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CONDUCTED
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BY

EDWARD L. NICHOLS,
ERNEST MERRITT, and FREDERICK BEDELL

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APRIL, 1912

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THE

PHYSICAL REVIEW.

ON THE FAMILY-TREE ARRANGEMENT OF THE ELEMENTS AND CALCULATION OF ATOMIC WEIGHTS ON THE CORPUSCULAR RING THEORY OF THE ATOM.

By Albert C. Crehore.

THE atomic theory which has been worked out in some detail by Sir J. J. Thomson[1] assumes that the atoms of matter consist of positive electrification enclosing a number of negatively electrified corpuscles exactly balancing the positive electrification when the atom is neutral. The corpuscles within mutually repel each other and seek positions of equilibrium as the result of the forces acting upon them. It has been shown that the corpuscles will distribute themselves as a result of these forces on concentric rings or shells within the sphere to attain equilibrium, the exact arrangement depending upon the number of corpuscles to be accommodated within it, and upon their velocities. The distribution, in rings, for equilibrium has been partially worked out for moderate velocities, but that in shells presents so many added difficulties that no complete solution is known.

Equilibrium figures closely related to those formed by the rotating electrons of the Thomson atom have been studied by Mayer in his well known experiments with floating magnets. In these experiments small magnetized needles were thrust through pieces of cork and floated upon water, each needle having its negative pole above the water. The needles mutually repel each other in much the same way as the corpuscles within the atom are supposed to, and each needle in the experiment represents a corpuscle in the atom. The positive electrification of the atom is represented by a positive magnetic pole suspended at some distance over the needles above the water. Under these conditions it has been observed

[1] J. J. Thomson, "On the Structure of the Atom," Philosophical Magazine, Vol. 7, p. 237, 1904.

that the needles arrange themselves as represented in Fig. 1. This gives the arrangement of the magnets from two to twenty, and the effect of the addition of a single magnet to the collection is well shown. If a single magnet is added there is in general only one point where it can be placed to preserve the equilibrium. It may go into the outer ring,

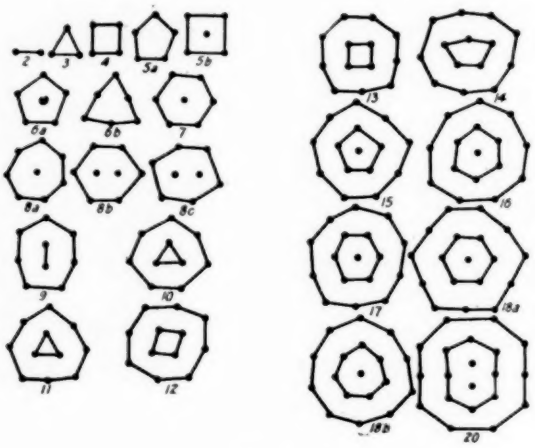


Fig. 1.

the next within it, or the third, etc., or may start to form a new ring at the center of the whole collection of concentric rings. The outer ring cannot receive it until each of the other rings has received its required allotment.[1]

The arrangements of the corpuscles for equilibrium have been approximately calculated for moderate velocities by Sir J. J. Thomson.[1] Table I. gives each configuration from one to one hundred corpuscles by intervals of unity. This table is represented graphically in Fig. 2. The broken line OA is obtained by plotting as abscissæ the total number of corpuscles comprising the system, and as ordinates the number of corpuscles in the first or outer ring, A, only. The lines B_2B, C_2C, D_2D, etc., represent in a similar manner the corpuscles in the second, third, fourth, etc., rings.

By adding the number of corpuscles in the first and second rings the line $OA + B$ is obtained, and by adding to this the third ring the line $OA + B + C$ is obtained. When the sixth ring is added the limit of the chart is reached and the whole collection or atom is represented by the line OG_1, and the broken lines B_1A, $C_1A + B$, etc., separate the atom into its component rings branching off from the atom at the points B_1, C_1,

[1] The Corpuscular Theory of Matter, page 109. J. J. Thomson, 1907.

D_1, etc. The portion between the line OB_1A and the straight line OG_1 represents the number of corpuscles left in the atom when the outer ring is removed, and the portion between $OC_1A + B$ and OG_1 represents the number left when the two outer rings are removed, and so on.

TABLE I.

| | | |

series of elements, and a curve to express this condition may be constructed by laying off as abscissæ the distances OB_2, OC_2, OD_2, etc., corresponding to ordinates one, two, three, respectively, these numbers representing the number of the ring in the atom. This curve is given in the upper portion of Fig. 3. The origin is taken, however, three units above the origin of the scale indicated at the left to avoid confusing the curve with the others on the same chart. The curve is a hyperbola having the equation,

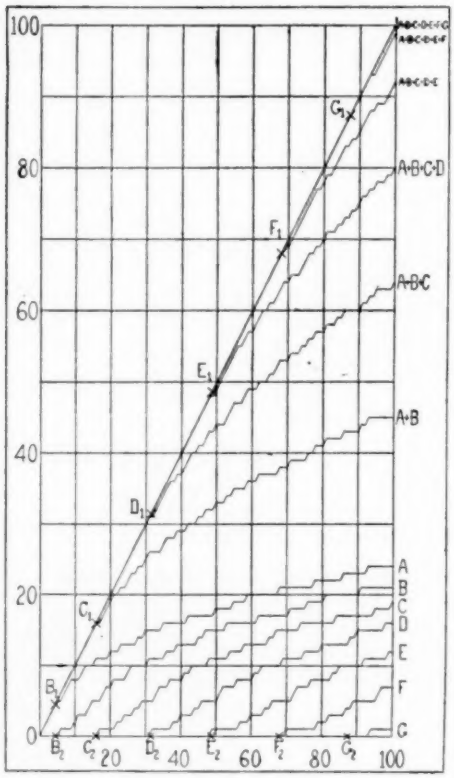


Fig. 2.

$$y^2 = .00233x^2 + .213x, \quad (a)$$

and the points B_2, C_2, etc., are marked to correspond with those in Fig. 2. The units on the vertical scale here correspond to the different series in the arrangement of the elements in the periodic law. The curve itself represents a group in the periodic arrangement of the elements, because the points on this curve correspond to those elements related to each other simply by the addition of successive outer rings.

This curve suggests a scheme for arranging graphically all of the elements as they appear in the table of Mendelejeff. Locate a series of points having abscissæ equal to the atomic weight, which is proportional to the number of corpuscles in the atom, and having ordinates equal to the number of the series in which the element is found in the table.

All of the elements in the 1911 International Table are thus represented in Fig. 3. The roman subscripts after the symbol of the element denote the group in which it is found in the table. Broken lines are drawn connecting elements in the same group. There are thus nine broken lines, each corresponding to one of the nine groups.

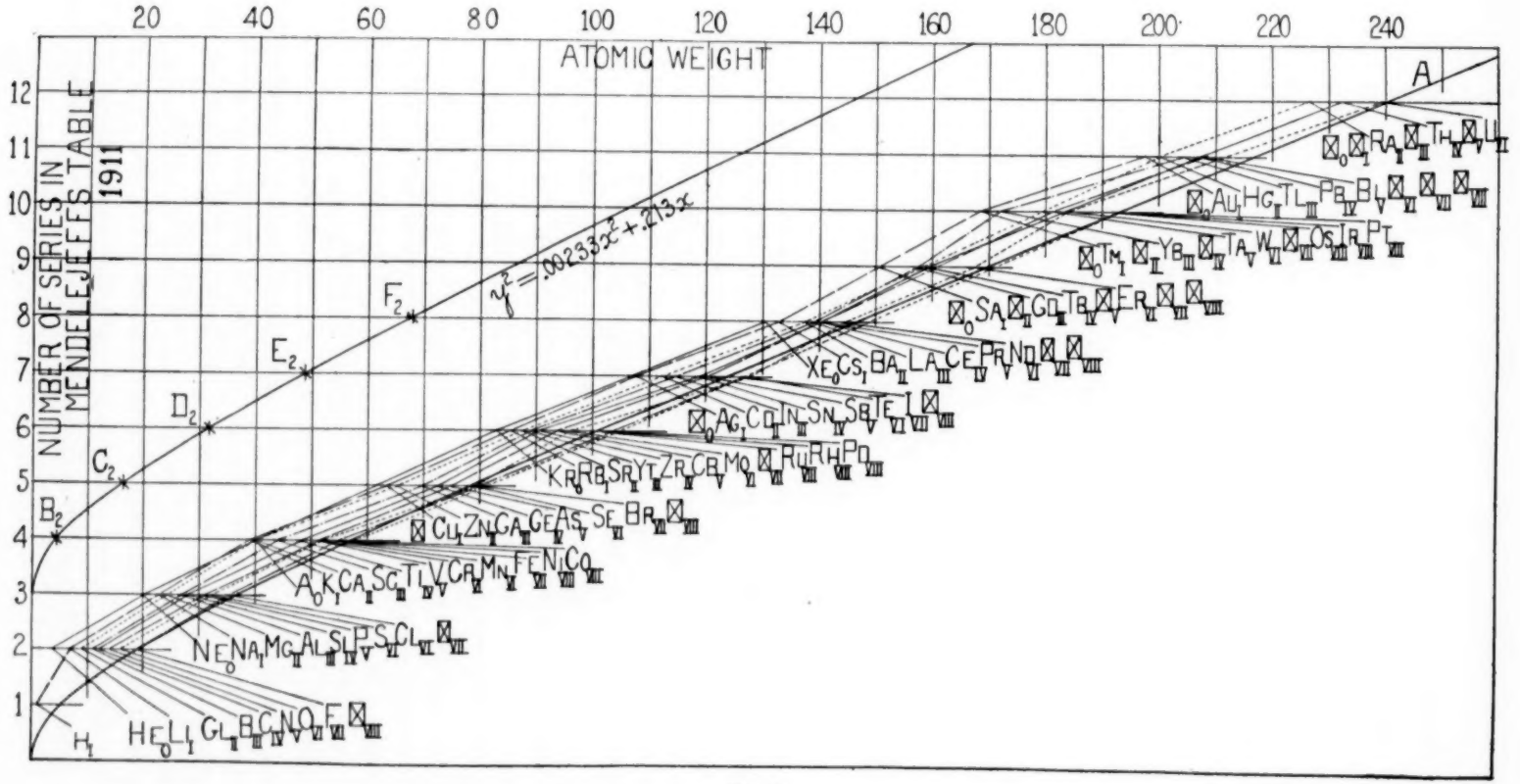

ATOMIC WEIGHT
NUMBER OF SERIES IN MENDELEJEFF'S TABLE 1911
$\nu_f^2 = .00233x^2 + .213x$
A
F_2
E_2
D_2
C_2
B_2
H_1

Fig. 3.

DETERMINATION OF THE NUMBER OF CORPUSCLES IN AN ATOM.

To compare the hyperbola B_2F_2, with these group-curves of the elements it is necessary to know the ratio of the number of corpuscles in the atom to the atomic weight; for it will be remembered that the abscissæ of the hyperbola represent the number of corpuscles in the atom, while those of the group curves of the elements represent atomic weights.

It is permissible, however, without knowing the number of corpuscles in the atom, to change the horizontal scale of the hyperbola to make it fit best the group curves of the elements. By trial it is found that when the abscissæ of the hyperbola are multiplied by the factor 1.16, and then plotted from O as origin, the hyperbola OA is obtained. This curve conforms very closely to the broken lines representing the seventh and eighth groups of the elements.

From these two curves it appears, for example, that the atom consisting of 68 corpuscles, the point F_2 on the curve in the fifth series, corresponds to an atomic weight of $68 \times 1.16 = 79$. An atomic weight of unity, therefore, corresponds to .863 of a corpuscle.

FAMILIES AND GROUPS IN TABLE I.

It is evident from Table I. that any combination of rings in a given series may be obtained from some combination in the preceding series by the addition of an outside ring. For example, consider the combination of 100 corpuscles made up of several rings, the outer ring having 24 corpuscles, the second 21, the third 19, etc. If the first ring of 24 is removed, the combination of 76 corpuscles with six rings is obtained, each ring having precisely the same number of corpuscles as the corresponding ring in the 100 collection. By removing the next ring of 21 corpuscles the collection of 54 is obtained, and by removing the third ring the 36 combination results. Removing the fourth gives 20, the fifth 8, and the sixth one corpuscle only. It makes no difference from which combination we start it is always possible to find a combination in the next lower order or series which has the identical distribution of corpuscles remaining after removing the outer ring.

Table II. is a repetition of Table I. omitting all figures except those giving the number of the series and the total number of corpuscles in the combination. Lines are drawn connecting certain particular numbers in adjoining series for the purpose of indicating those combinations which are identical in the distribution of corpuscles in each ring, when the outer ring in the lower series is omitted. For example, starting with the number 88 and following the lines upward, we pass to 65, 45, 28, 14 and finally to 4, losing an outside ring each time. In the reverse

direction, however, starting with 4 and following the lines downward, we come to 14, at which number the way divides into two alternative paths, the one leading to the numbers 27, 44, 64 and 87, and the other to 28, 45, 65 and 88.

TABLE II.

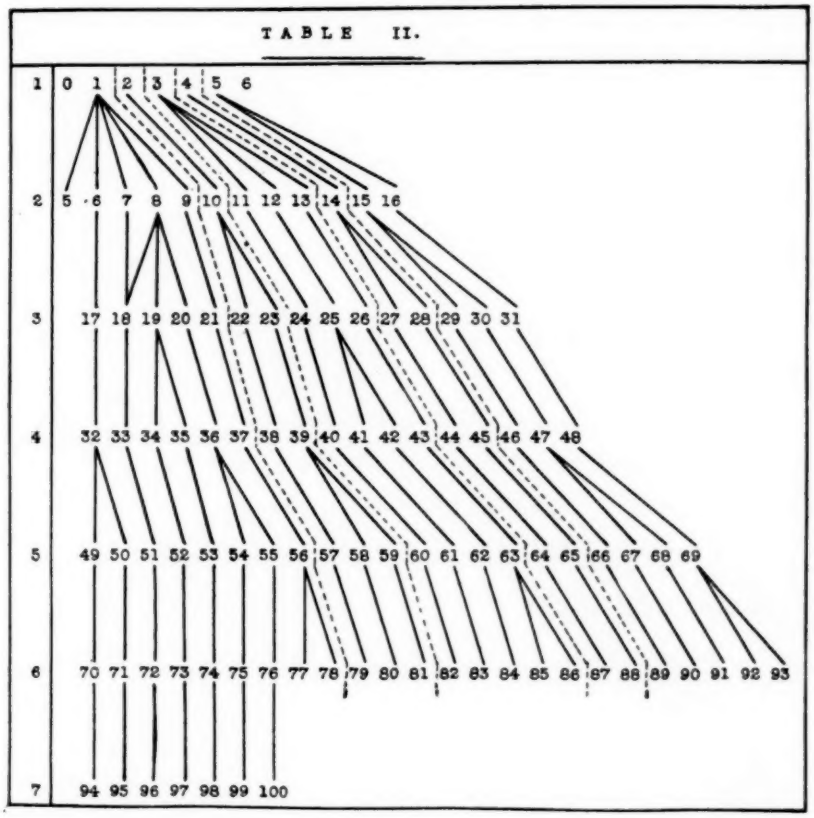


Table II. is represented graphically in Fig. 4, the abscissæ being the total number of corpuscles in the collection, and the corresponding ordinates the number of the series in the table set opposite the horizontal rows of numbers.

The very close resemblance of Fig. 4 to the chart representing the elements, Fig. 3, is too striking to be the result of any chance. It deserves the closest study, but perhaps the most important result is obtained from the suggestion contained in this figure that the proper way to arrange a table of the elements is in tree formation having branches leading away from a common origin. The farther we proceed from this origin the more numerous are the branches. It seems clear that the old way of arranging the elements in Mendelejeff's table, where each

group of elements occupies a column of its own having no alternative column as we descend from series to series, must be incomplete. A need for more spaces in the table to accommodate elements recently suspected to exist has already been felt, and the following family-tree arrangement, based exactly upon Table II. and Fig. 4, supplies this need, and besides gives an arrangement of the elements in tabular form which agrees exactly with the corpuscular ring theory.

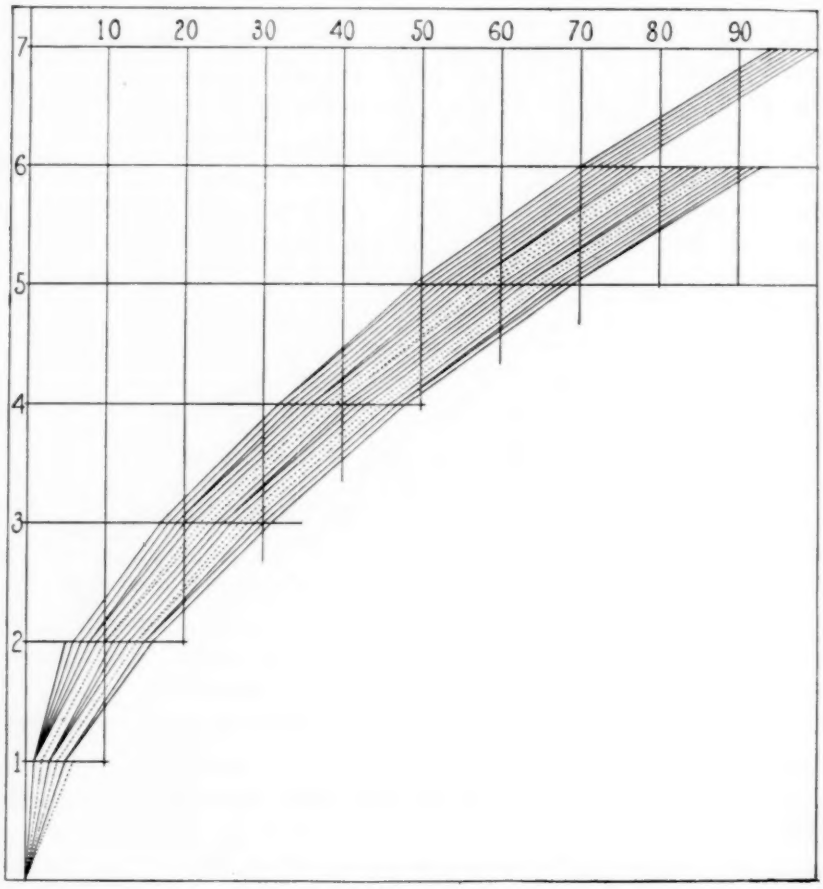
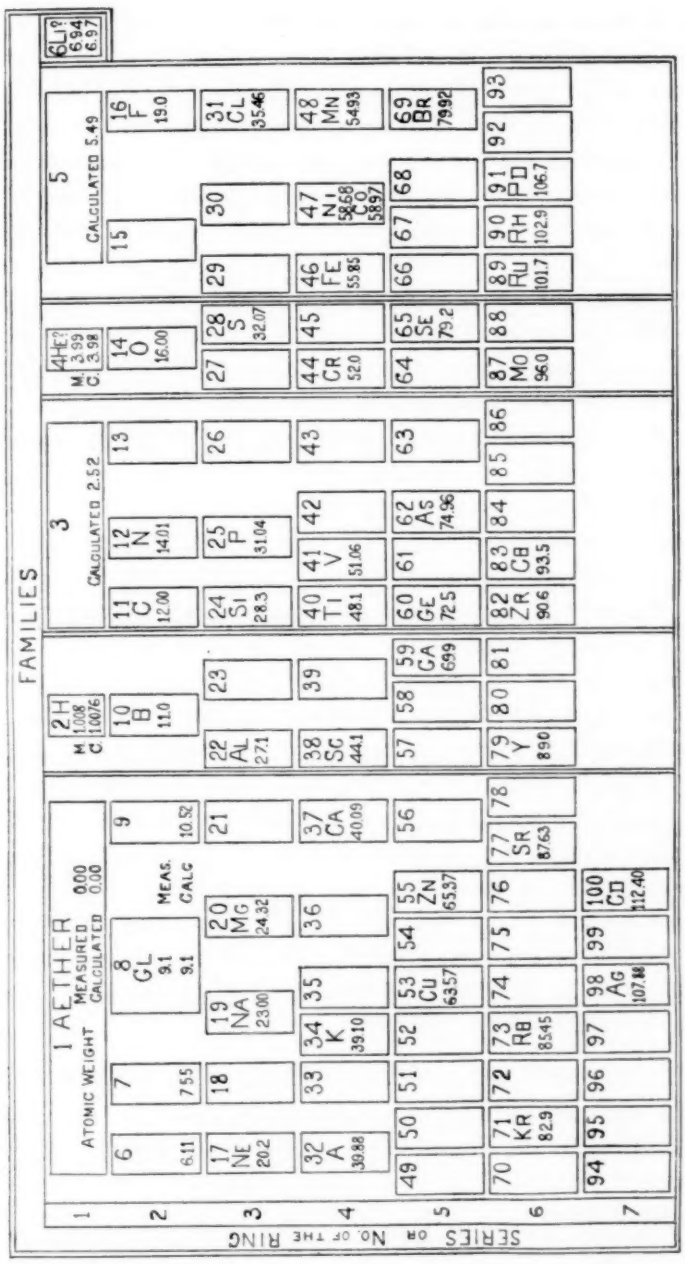


Fig. 4.

An arrangement of squares numbered consecutively from 1 to 100 based exactly upon Table II. and Fig. 4 is presented in Table III. In Fig. 4 five families of lines starting from the first five corpuscles composing the simpler atoms are clearly marked. At the upper limit of the figure there are in all 24 paths, and the upper nine paths, indicated by full lines, all emanate from one corpuscle. The next three, indi-

TABLE III.



cated by dotted lines, start from the group of two corpuscles. Below these the next six full lines emanate from the group of three corpuscles. There are then two dotted lines starting at the group of four corpuscles, and finally a group of five full lines from the group of five corpuscles.

Each of these families is separated very completely from each of the others, as it is impossible to pass from one family to another by any of the lines in Table II. For this reason the vertical double lines are used in Table III. to separate the five families completely.

Although there are but five separate families, yet it is easy to accommodate the nine groups of the elements in Mendelejeff's table. The zero, first and second Mendelejeff groups find their places in the first family, the third group in the second family, the fourth and fifth groups in the third family, the sixth group in the fourth family, and the seventh and eighth groups in the fifth family. In some of the families it is not so difficult to assign elements to their proper squares because there is not much choice. This is particularly true in the fourth family in which there are but two branches in Fig. 4. Oxygen is assigned to the 14th square, sulphur to the 28th, chromium to the 45th, selenium to the 65th, and molybdenum to the 88th square.

The reason for assigning oxygen to the 14th square is because its atomic weight, 16 multiplied by the factor .863 previously determined to be the ratio of the number of corpuscles in the atom to its atomic weight, gives the number 13.8, nearly equal to 14, being only 1.3 per cent. less than 14. Similarly, multiplying the atomic weight of sulphur, 32.07, by .863 we obtain 27.6, a quantity 1.4 per cent. less than 28, which is one of the two numbers immediately under 14 in the 4-family. At this point it is difficult to decide which of the two alternative paths to take because the number 27.6 falls between 27 and 28. By considering the next element in the group, chromium, however, the number 44.8 comes nearer to 45 than to 44, and this consideration places sulphur in the 28th instead of the 27th square. The next two elements, selenium and molybdenum, do not agree as well with the whole numbers 65 and 88, but when all is considered it seems truly remarkable that there is such a close agreement as we find.

A Second Graphical Representation of Table II.

If we compare the two outside paths in Table II. the one in family 1 terminating in the number 94, and the other family 5 terminating in

Lowest Curve.	Differences or Number of Paths.	Highest Curve.
93		94
69	24	70
48	21	49
31	17	32
16	15	17
5	11	6
0	5	1

the number 93, it appears that the differences in these two columns are exactly the same.

The same facts are seen in Fig. 4, for, if the lowest curve of the set, beginning at zero and terminating at 93, is moved as a whole one unit to the right and also one unit upward it exactly coincides with the curve beginning with 1 and terminating with 94. This suggests the graphical method of representing the same thing in Fig. 5. If all the lines between

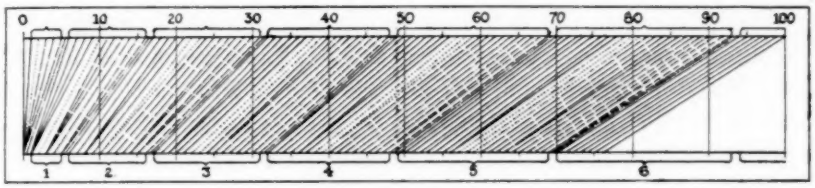


Fig. 5.

the first and second groups in Fig. 4 are lowered one unit without disturbing the horizontal distances, so that all lines on the one unit line will be dropped vertically downwards to the zero line, it is observed that each group of lines exactly fits in with the adjacent groups, adjoining lines being parallel to each other at one unit distance apart.

Fig. 5 is thus a reproduction of Fig. 4 with this difference, that the curves in Fig. 5 may be considered to be drawn upon the surface of a cylinder, each curve following the path of a screw thread having variable pitch, greater as we descend, each turn of the thread corresponding to a new series of the elements. The circumference of the cylinder is equal to one unit on the vertical scale in Fig. 4, that is, to one series. The axis of the cylinder is parallel to the horizontal axis in Fig. 4 which corresponds to the number of corpuscles in the atom.

In this manner all of the atomic weights, or of the number of corpuscles, are brought into one straight line indicated at the top of Fig. 5, and the relationship of any one element to any other may be seen by following the path drawn through that number. Each of the five families is distinguished by a different kind of line, the first family by a full line, the second by a fine dotted line, the third by long dashes, etc.

Outline of Method of Calculating Atomic Weights.

The theory of gravitation proposed by the writer[1] has thrown light on a possible method of calculating atomic weights. The indication is that the gravitational attraction of the earth for the atoms of the different

[1] Electrical World, Feb. 10 and 24, 1912. These papers, with the present one, will be included in a series of papers by the writer about to be issued.

elements may be calculated by adding together the attractions and repulsions of the earth for the constituent parts of each atom, namely, the positive electricity and the individual negative corpuscles revolving in rings within the positive electricity. At this time only an outline of the method can be given, but it seems as though efforts in this direction will finally be successful.

Let P denote the attraction of the earth for a charge of positive electricity equal and opposite to the charge carried by one corpuscle or electron, referred to the weight of the hydrogen atom as a unit, and p the total number of corpuscles in the atom. The attraction of the earth for the positive electricity of any atom is then $= Pp$.

To determine the attractions and repulsions upon the negative corpuscles is more difficult because a knowledge of the angular velocity of the corpuscle is required before we can tell which formula to apply in deriving the attraction. The atomic weights themselves may, however, form some guide as to their velocities by finding that formula which gives results in closest agreement with the atomic weight.

Assuming that the angular velocities of the corpuscles in the rings of the different atoms are equal, the values of atomic weights in the only cases as yet calculated come as near as would be expected to the measured atomic weights. Besides a knowledge of the angular velocity, it is also necessary to know the positions of the different rings of corpuscles within the atom. These positions have never been calculated so far as I am aware. It seems worth while in the absence of an accurate solution near the velocity of light to obtain the positions of the corpuscles within an atom on the basis that they are moving slowly compared to the velocity of light, as being possible of calculation for the simpler elements.

An illustration of the method is here given on the basis of equality of angular velocity, and determination of the positions of the corpuscles as if at rest.

The gravitational attraction upon the positive sphere of the atom is given by the formula[1]

$$F_p = \frac{\mu q'^2}{6} \frac{ee'}{r^2}, \tag{1}$$

and the force upon a revolving negative corpuscle under this hypothesis by the formula

$$F_n = \frac{\mu q'^2}{6} \frac{(a' - a)}{a'} \frac{ee'}{r^2}. \tag{2}$$

The ratio of the force upon a revolving corpuscle to that upon a positive charge of equal amount is the ratio of (2) to (1), namely,

[1] For the derivation of (1) and (2), see Electrical World, Feb. 24, 1912.

$$\frac{F_n}{F_p} = \frac{a' - a}{a'}.$$

If n_1, n_2, n_3, etc., denote the numbers of corpuscles in rings in the atom, number one being the outside ring, the force upon one corpuscle in the outside ring is

$$\frac{a_1 - a}{a_1} P. \tag{3}$$

This is positive and denotes an attraction when a_1 is greater than a, and is a repulsion when a_1 is less than a. a_1 denotes the radius of the outside ring of corpuscles in the atom of the element being attracted, and a the radius of a ring in an average or equivalent atom in the earth. By this is meant the radius of the ring of corpuscles in a hypothetical atom, which has one ring only and represents the equivalent of all the atoms composing the earth. If the earth is supposed to be composed of such hypothetical atoms only, the total number of the corpuscles contained therein being the same as the total number of corpuscles in the atoms making up the earth, then the atomic weights of the elements would not be altered. The determination of the equivalent radius a and of the force P upon a positive charge equal and opposite to that on a corpuscle may be accomplished as follows. If A denotes the atomic weight of any element we obtain by applying formula (1) to the positive electrification and (2) to the corpuscles,

$$A = pP + P\frac{a_1 - a}{a_1} + P\frac{a_2 - a}{a_2} + \cdots. \tag{4}$$

The first term in the right member denotes the attraction of the earth for the positive electricity, and added to this is one term for each corpuscle within the positive sphere. The quantities a and P are constant because they depend upon the constitution of the earth; and a_1, a_2, etc., denote the radii of the orbits within the positive sphere of the several corpuscles composing the neutral atom. It is not strictly true that the corpuscles arrange themselves in rings which are exact circles when they form equilibrium figures within the positive sphere, and to apply this method more exactly it will be necessary to locate each corpuscle. If it is assumed, however, that they form circular rings the equation (4) may be written

$$A = pP + n_1P\frac{a_1 - a}{a_1} + n_2P\frac{a_2 - a}{a_2} + \cdots. \tag{5}$$

This applies to the case where there is no corpuscle at the center of the sphere, the first term being due to the positive electricity, the second to

the outside ring and third to the second ring next to the outside, and so on. When there is one corpuscle at the center as in the first family, the repulsion upon it is equal to P, and the first term in the right-hand member of (5) becomes $(p - 1)P$.

Applying the formula to the case of two elements, say hydrogen and glucinum, and assuming that there are two corpuscles in the hydrogen and eight in the glucinum atom, the latter consisting of a ring of seven and one corpuscle at the center of the sphere, we find for hydrogen

$$A = 2P + 2P\frac{.5 - a}{.5} = 1.008 \tag{6}$$

and for glucinum

$$A = (8 - 1)P + 7P\frac{1.19 - a}{1.19} = 9.1. \tag{7}$$

The values of the radii of the rings for hydrogen and for glucinum are referred to the radius of the positive sphere of the hydrogen atom as unity, and the volume of the positive sphere is taken as proportional to the total charge in the atom. The radius of the ring of two corpuscles in hydrogen is .5 or one half the radius of the positive sphere, which is taken as unity. The radius of the ring in glucinum is .747 of the radius of its sphere of positive electricity, but this radius is 1.59 times the radius of the hydrogen sphere, and the ring is therefore $.747 \times 1.59 = 1.19$ times the radius of the positive hydrogen sphere. Dividing (7) by (6) P cancels, and we find that $a = .731$ and $P = .935$.

The formula by which the radius of the outside ring of corpuscles in the atom may be calculated for slow velocities is

$$\left(\frac{a}{b}\right)^3 = \frac{1}{p}\left(\frac{S_n}{4} + m\right),$$

where a = the radius of the outside ring, b = the radius of the sphere of positive electricity, p = the total number of corpuscles in the neutral atom, n = the number of corpuscles in the outside ring, and $m = p - n$ = the remaining corpuscles in rings of smaller radius than a, and in some cases one corpuscle at the center of the sphere.

$$S_n = \operatorname{cosec}\frac{\pi}{n} + \operatorname{cosec} 2\frac{\pi}{n} + \operatorname{cosec} 3\frac{\pi}{n} + \cdots \operatorname{cosec}(n - 1)\frac{\pi}{n}.$$

The formula is exact when there is not more than one ring of corpuscles, and approximate for more than one ring.

If it is assumed that the volume of the positive electricity is proportional to the total number of corpuscles contained therein, a relation

which agrees with experiment as nearly as could be expected, and that the radius of the positive electricity in hydrogen is a unit of reference, the radius of the positive sphere for any other atom is

$$b = \left(\frac{p}{2}\right)^{\frac{1}{3}},$$

on the assumption that there are just two corpuscles in the hydrogen atom. This is the only number of corpuscles for the hydrogen atom which seems to give the correct atomic weights.

Table IV. gives the atomic weights for each combination of corpuscles as calculated up to but not including boron, which is the first combination having two rings of corpuscles, assuming that $a = .73$ for the earth, and $P = .933$. It is remarkable that the measured atomic weights of hydrogen, helium, lithium and glucinum coincide almost exactly with the weights calculated for the even numbers of corpuscles, $p = 2, 4, 6,$ and 8 respectively, and that carbon is the only element in the second series which has an odd number of corpuscles.

The table has been extended to include all of the second series of elements; but, since the position of the inside ring of corpuscles is not known, the atomic weights have not been calculated as before but assumed to agree with the experimental values, and the radius, c, of the inside ring calculated.

The process of calculating the radii of the rings independently for equilibrium involves much labor, and is being carried out at the present time for two rings only. When there are three or more rings the difficulties rapidly increase.

If it turns out to be true that atomic weights may be calculated in this way, it will be difficult to avoid the conclusion that the fundamental assumptions upon which the calculation is based are nearly correct. One of these assumptions is that the rings in all the atoms which have been calculated revolve with the same angular velocity. It will be interesting to find the value of this velocity. Perhaps some cause which operates to synchronize the electrons of the atoms will be found.

The calculation of the combinations of corpuscles as in Table I. should certainly be carried far enough to include the heaviest element, uranium. The present form of Mendelejeff's table has $12 \times 9 = 108$ squares, and in the eighth group more than one element is placed in one square. With the new arrangement of the family-tree there will be more than 100 additional squares. There is no doubt that all of the squares will never be filled with known stable elements. Many of them may, however, be assigned to certain squares as existing in a temporary state for a brief

time only, the combination serving as a transition station not sufficiently stable for long continued existence.

TABLE IV.

Atomic Weights.

No.	Element.	p	Rings.	a/b	c/b	c/a	a	b	c	Atomic Weights.	
										Calculated.	Observed.
1	Æther....	1	—	—	—	—	—	0.794	—	0.000	0.000
2	Hydrogen.	2	2–0	.5000	—	—	.500	1.00	—	1.0076	1.008
3	"	3	3–0	.5773	—	—	.661	1.145	—	2.52	—
4	Helium ...	4	4–0	.6208	—	—	.781	1.26	—	3.98	3.99
5	"	5	5–0	.6505	—	—	.885	1.36	—	5.49	—
6	Lithium ..	6	6–0	.6726	—	—	.97	1.443	—	6.97	6.94
7	"	7	6–1	.739	—	—	1.122	1.518	—	7.55	—
8	Glucinum .	8	7–1	.747	—	—	1.19	1.59	—	9.1	9.1
9	"	9	8–1	.751	—	—	1.24	1.65	—	10.52	—
10	Boron	10	8–2	.783	.222	.284	1.34	1.71	.38	—	11.0
11	Carbon ...	11	8–3	.808+	.243	.300	1.43	1.77	.43	—	12.00
12	Nitrogen..	12	9–3	.808−	.265	.328	1.47	1.82	.482	—	14.01
13	Oxygen...	14	10–4	.825	.245	.297	1.58	1.91	.469	—	16.00
14	Fluorine ..	16	11–5	.838	.266	.317	1.68	2.00	.532	—	19.0

p = total number of corpuscles in the atom, b = radius of sphere of positive electricity, a = radius of outside ring of corpuscles, and c = radius of inside ring each referred to the radius of the positive sphere of hydrogen as unity. $b = (p/2)^{\frac{1}{3}}$, $P = .933$, and $a = .73$ for the earth.

THE ÆTHER.

The first case in the table, which is that of one corpuscle only at the center of the positive sphere having a charge equal to but of opposite sign from the corpuscle, is perhaps the most interesting of all. The formula for this case where there are no rings reduces to

$$A = (p - 1)P$$

and since $p = 1$, the formula makes the atomic weight absolutely equal to zero. That is to say, if such an element exists the earth and sun and planets have absolutely no attraction for it. It would be consequently uniformly distributed throughout space having no tendency to condense anywhere. In this respect it resembles the æther, the medium heretofore supposed to transmit light and electromagnetic waves through space.

It is true that some advocates of the relativity principle deny the existence of any such medium, but it will be nevertheless interesting to put to the test the proposed structure of the æther to ascertain whether it will not by its properties perform all of the functions that are required of it.

Æther according to these indications would be located in the first square of the first series in Mendelejeff's table, and there can be no further use for a zero series as the table is now published. Mendelejeff himself has advocated placing the æther in the table, but I believe that he does not place it in the first series, and also attributes a very small atomic weight to it, not absolutely zero.

COMPARATIVE STUDIES OF MAGNETIC PHENOMENA. II.

RELATION BETWEEN THE JOULE EFFECT AND PERMEABILITY AND INDUCTION IN THE SAME SPECIMENS OF STEEL.[1]

BY S. R. WILLIAMS.

INTRODUCTION.

A SURVEY of the work which has been done in investigating the the varied phenomena of ferromagnetism indicates that not enough comparative work has been produced. That is, the relations between several magnetic effects have not been established in the same identical specimens where in many cases such comparisons were possible. For instance, one writer may compare magnetic change in length of a steel rod with the permeability while another inquires into the relation between the carbon content and the Villari effect in an entirely different specimen of steel. It is evident that there can be little or no coördination of such material, since one specimen of steel varies from another in degree of hardness, the previous heat and magnetic treatment, to say nothing of the carbon content itself, and other alloys and impurities. If the carbon content, induction, permeability, the Joule and other magnetostrictive effects, the Villari effect, the Kerr magnetic effect, etc., could all be studied in the same specimens, such a comparison would make an important contribution to our knowledge of the subject of ferromagnetism and especially to any attempt to get at a working hypothesis of ferromagnetism. This latter point cannot be too strongly emphasized, because if we had a working theory[2] which would coördinate such diverse phenomena as the magnetostrictive effects with the Kerr magnetic effect or the Faraday effect in thin films, it would furnish a great impetus to the work now being done in this field. With this phase of the subject in mind I have made a collection of rods and tubes of various ferromagnetic substances in which I propose to study the relation of all the different magnetic effects possible to be investigated in specimens of the rod or tube form.

In a former paper[3] I have pointed out that there are indications that

[1] Presented before the meeting of the American Physical Society in Washington, Dec. 29, 1911.

[2] Knowlton, Ter. Mag., Vol. 15, No. 1, 1910.

[3] PHYS. REV., p. 46, Vol. 34, Jan., 1912.

the Joule magnetostrictive effect may well be studied in comparison with other magnetic effects, and so in a series of studies I wish to discuss some of these relations which I have found between the Joule and other magnetic effects. The present paper deals with the relation between the Joule magnetostrictive effect, and the permeability and induction for corresponding field strengths in the same specimens of steel. In a paper[1] on the "Comparison of the Joule and Wiedemann Magnetostrictive Effects in Steel Tubes" I showed that the permeability seemed to play an important rôle in these two phenomena. It therefore seemed worth while to study in greater detail the relation between the Joule effect and the quantities involved in the relation $B = \mu H$. To this will be added data on the carbon content.

The Joule Effect.

One of the difficulties in measuring the change in length of any ferromagnetic substance due to a magnetic field is the masking of the effect by changes in length due to variation in temperature. By some investigators this has been overcome by compensation devices, by others, temperature changes have been reduced to a minimum by inclosing the specimen in a jacket whose temperature could be maintained fairly constant by allowing water to flow through the system.

To make a series of observations over any great range of field strengths without some temperature control is impossible with the ordinary telescope and scale method, because it takes too long to adjust the field strength and then to read the changes in length which have occurred. In previous work on the Joule effect I found that if in varying the magnetic field the changes were produced quickly enough, the changes due to temperature did not appear in time to vitiate the results. It therefore occurred to me that if some automatic method could be devised for quickly recording the changes in length with the corresponding field strength, it would be a great advantage, as we would then eliminate heat effects by obtaining the desired results before disturbances due to temperature appeared. That this has been accomplished in a very satisfactory way is shown, I think, in the results given and from the observed fact that after a complete series of readings had been obtained by this method some time elapsed before temperature changes manifested themselves.

The general method of measuring the Joule effect is to place the specimen experimented upon along the axis of a solenoid whose ampere-turns are known, and measure by means of some multiplying device the changes in length which occur as the field in the solenoid is varied from zero up-

[1] Phys. Rev., p. 293, Febr., 1911.

wards. Schematically this is shown in Fig. 1, where as a multiplier of the effect I have used a system of lever and mirror more fully described in a previous paper.[1] This system was found very satisfactory and would repeat the small details of any particular graph with surprising faithfulness, although given a thorough shaking up each time in demagnetizing with a decreasing alternating current. In determining the multiplication the table of a dividing engine was connected to the stirrup where the lower end of rod was usually attached. This point could be moved through a measured distance by means of the dividing engine's screw while the deflections of the mirror were observed at the same timeby a scale and telescope. These observations could be repeated to one part in ten thousand. In connection with this scheme of multiplying the effect I have devised a simple arrangement for photographing the deflections of the mirror and doing it in a very short space of time. This is shown in Fig. 2. Mr. T. S. Towle, an advanced student in the department, worked out the mechanical details of the arrangement and I am much indebted to him for the success of the method.

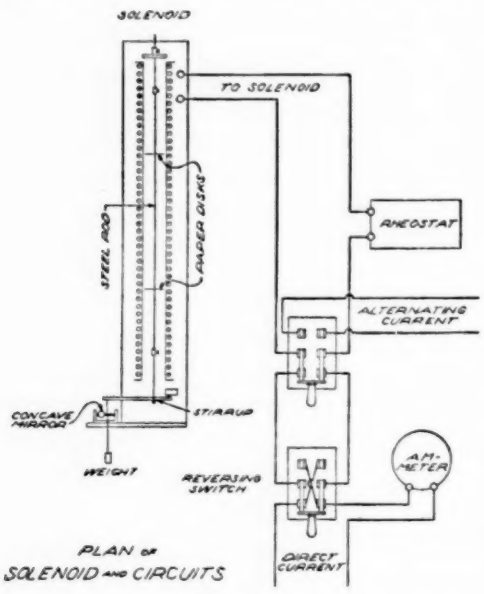


Fig. 1.

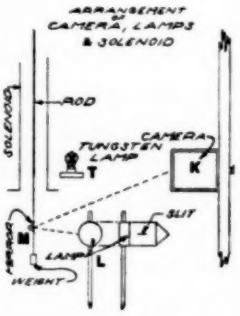


Fig. 2.

MR. TOWLE'S EXPERIMENTS.

The light from a straight filament bung-hole lamp, *L*, fell upon the concave mirror, *M*, which formed an image of the filament on the camera, *K*. The lens was replaced by a diaphragm containing a long narrow slit. The length of the image of the filament was at right angle to the slit so that only a spot of light passed through. As the rod changed its length the mirror was tilted by means of the lever attached to the lower end of the rod and the image of the lamp filament moved up or down the slit

[1] PHYS. REV., p. 285, Febr., 1911.

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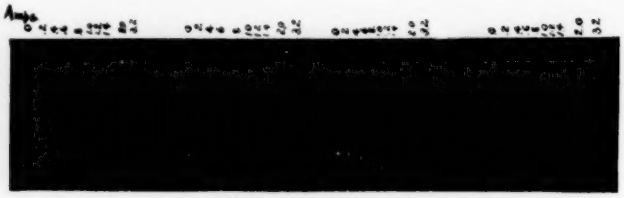


ROD 1. FILM DISTANCE 414 CM.

ROD 2. FILM DISTANCE 155 CM.

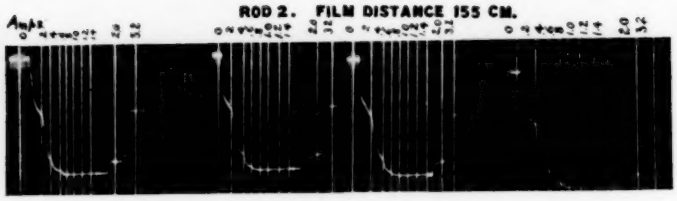
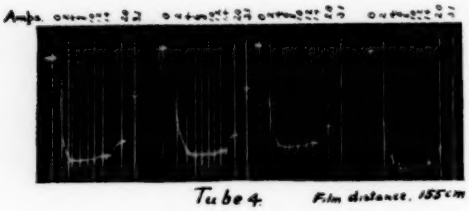


Fig. 4.

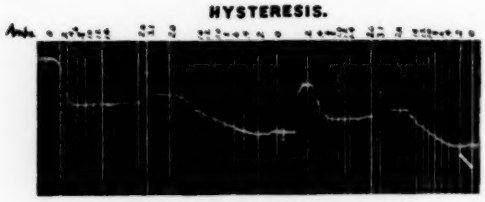
Rod 3 Film distance, 155 cm



Tube 4. Film distance, 155 cm

Fig. 5

HYSTERESIS.



ROD 3 FILM DISTANCE 155 CM.

TUBE 4 FILM DISTANCE 155 CM.

Fig. 6.

S. R. WILLIAMS.

PLATE I.

of the camera depending upon whether the rod shortened or lengthened. Immediately behind the slit was a photographic film which could be moved along by means of a suitable mechanism which is indicated in Fig.

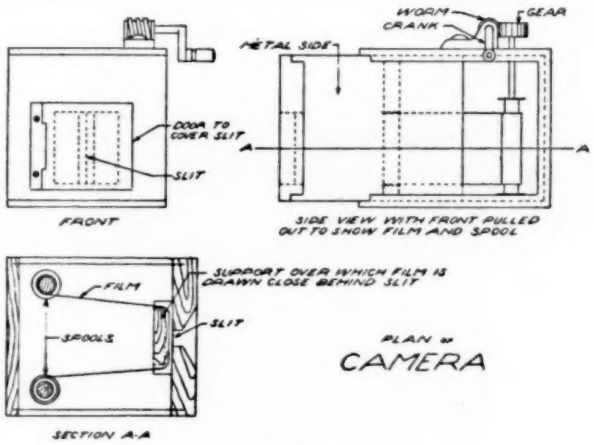


Fig. 3.

3. The graphs obtained by this means are shown in Figs. 4, 5 and 6. The actual deflection of the spot of light can be measured from these graphs and knowing the film distance from the mirror and the multiplying power of the levers, the elongation of the rods could be determined. For recording the field strengths corresponding to a certain elongation of the rod, a tungsten lamp, *T*, Fig. 2, was placed in front at some distance

Rod 1.

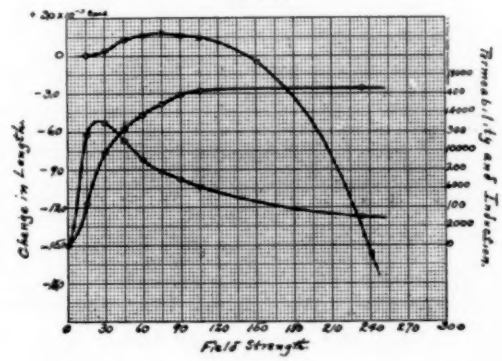


Fig. 7.

away and above the camera. When the current in the solenoid reached a certain known value this lamp was flashed, making an image of the slit as a marker across the photographic film. Knowing the ampere turns

of the solenoid, the field strength for each flash could easily be determined. The values of the deflections are taken at the intersections of the curve and the straight line serving as a marker, because at these points we know the field strengths. The method of taking observations was as

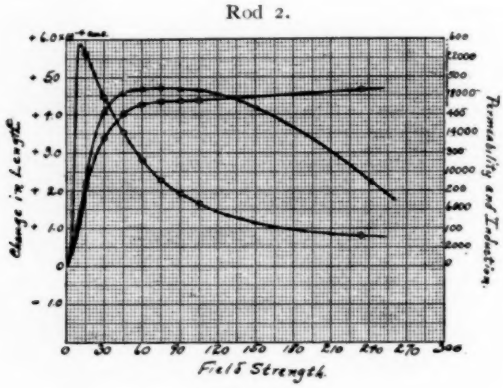


Fig. 8.

follows: A rod was placed in position and magnetized a few times and then demagnetized by sending a decreasing alternating current through the solenoid. The system was then allowed to stand for some little time

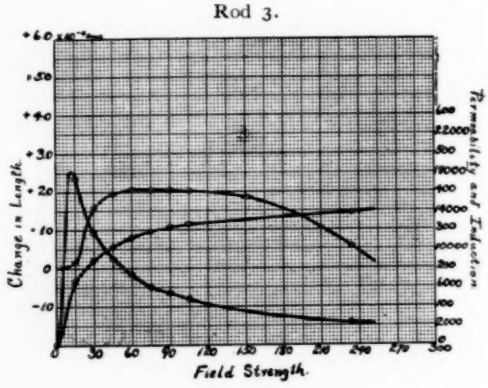


Fig. 9.

until things became settled. At a signal from the operator reading the ammeter and controlling the rheostat in series with the solenoid the assistant would commence to turn the crank operating the motion of the film. The operator at the ammeter would immediately begin to increase the current in the solenoid from zero upwards by means of a liquid rheostat which could be varied steadily and not by steps. As the ammeter pointer moved over the points marked 0, .2, .4, .6, etc., amperes,

the lamp, T, would be flashed by means of a telegraph key in series with it. In this way a whole series of readings could be obtained in a few moments of time and on development of the plates the accurate measurement of the deflections could be made at one's pleasure. The rate of moving the film does not matter because at the instant the lamp flashes

Tube 4.

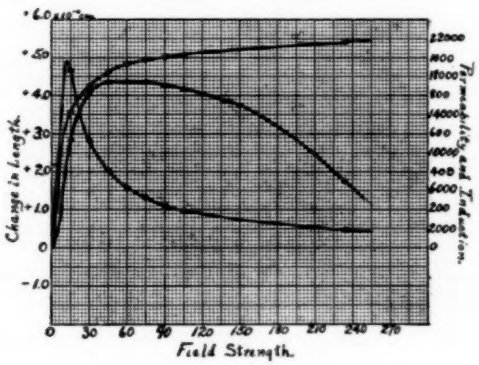


Fig. 10.

the point of intersection of the curve and the line marking the field strength gives the deflection for that particular field strength for which the lamp was flashed. The points are then plotted as shown in Figs. 7, 8, 9 and 10. The film might be operated by clockwork instead of by an assistant, but it was found so convenient to have an assistant that this plan was not carried out.

Description of Rods and Tube Used.

Three rods and one tube of steel were investigated with respect to the Joule effect, permeability and induction, and were numbered 1, 2, 3 and 4 respectively. Their dimensions are as follows:

Table I.

	No. 1.	No. 2.	No. 3.	No. 4.
Length.....	80 cm.	80 cm.	80 cm.	80 cm.
Diameters.....	.6386 cm.	.6300 cm.	.5800 cm.	O.D. .6409 cm. I.D. .5488 cm.

Specimens in the rod or tube form were studied because it would seem that a greater number of comparisons could be carried out on samples in that shape and at the same time they would be more convenient for

various heat treatments. Again it seemed that a rod 80 cm. long would give a better idea of the character of any particular specimen of steel than a short one, say 10 cm. long. Pierce[1] has shown that the hardness varies greatly along any given bar, hence if ten centimeters from one end of a long rod was compared with ten centimeters from the other end, very discordant results might arise because the hardness plays an important part in the magnetic properties of steel. For a given carbon content the behavior of a long rod may more safely be compared with the behavior of another long rod since one takes the summation effect of the whole. This applies particularly to a change in length study.

Fig. 11 shows the brass extensions used at both ends of the rods so that they would extend beyond the ends of the solenoid, whose length was 100 cm. These extensions also served as support for the upper end of rod and as a connection for the levers at the lower.

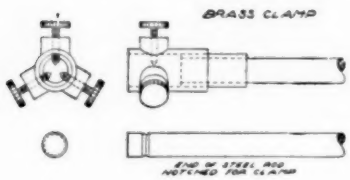


Fig. 11.

Figs. 4 and 5 show sets of graphs for the rods 1, 2 and 3, and the tube 4 respectively. In each set are four graphs and the curves of change in length in Figs. 7, 8, 9 and 10 are the mean of these four graphs for each specimen. The four graphs in each set show the deflections when field is first down then up, then down and finally up consecutively in the solenoid, demagnetization occurring between each reversal.

I am much indebted to Mr. W. N. Crafts, president of the Crucible Steel Forge Co., Cleveland, Ohio, for the rods 1 and 2 and also wish to express my appreciation of the many valuable suggestions received from him regarding the production and heat treatments of steel. The effect of heat on these specimens of steel will be studied later. Through the courtesy of the chemical department, Mr. Earl Rice, an advanced student, determined the carbon content of the specimens by the chromic acid method. I take this opportunity to express my thanks for this favor. The values found are as follows:

TABLE II.

Rod No.	Carbon Content. Per Cent.
1	1.05
2	0.20
3	1.04
4	0.32

[1] Pierce, Proc. Amer. Acad., Vol. 46, No. 8, Oct., 1910.

In addition to the results obtained from my own observations I have calculated and plotted the results obtained by other observers. In Fig. 12

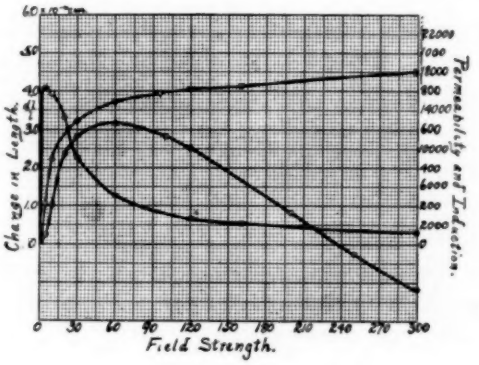


Fig. 12.

are shown the results obtained by Nagaoka and Honda[1] on a specimen of steel, ten centimeters long. I add the curves for the sake of showing how they corroborate the general results of my own work.

Hysteresis.

This rapid method of obtaining results is particularly applicable to a study of the hysteresis occurring in the change in length due to a magnetic field. Fig. 6 shows graphs of the hysteresis found in the length change of rod 3 and tube 4, while Fig. 13 is a plot of the results obtained from Fig. 6.

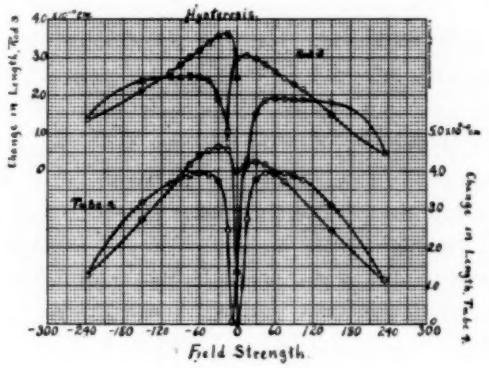


Fig. 13.

Discussion of Results.

The photographs obtained from rod 1, Fig. 4, were taken at a film distance of 4.14 meters, nearly three times as far away as that for rods

[1] Nagaoka and Honda, Phil. Mag., p. 45, July, 1902.

2 and 3 and tube 4. Even at this large film distance the initial elongation is very small. When working at the film distance used for the other rods I could not definitely detect this initial elongation and so appeared to have a specimen whose behavior was similar to that found by Bidwell[1] in a sample of steel. Others have observed the same phenomenon in steel, viz., the absence of an initial lengthening. From several cases of this sort I have been led to believe that if the magnifying power of the device for showing the Joule effect had been great enough in the case of other observers, they too would have found an initial lengthening for steel. The magnifying power used for rod 1 was about 24,000. If the assumption just made holds good it points to a uniformity in the behavior of iron and steel in the Joule effect, viz., an increase in length for small field strengths and a shortening for strong.[2]

In Figs. 7, 8, 9, 10 and 11, I have compared the curves for the Joule effect, permeability and induction. What seems to me to be of importance in the relation of these curves is the fact that the maximum elongation occurs at a field strength not far from that at which the knee of the induction curve occurs and necessarily the maximum permeability occurs in all cases at a lower but related field strength, as shown in previous work.

In Winkelmann's Hanbduch der Physik,[3] Auerbach has given an excellent resumé of three important theories of magnetism, (1) Scheidungshypothese, (2) Richtungshypothese, (3) Wirbelhypothese. The relation just spoken of between the change of length and the induction is, in my opinion, a strong argument in favor of the second hypothesis, viz., the orientation theory. As I pointed out in a previous paper[4] the magnetostrictive effects are easily explained from the standpoint of the orientation of elongated elementary magnets. If these elementary particles possess permeability, as I have assumed, then their rotation will influence the induction and permeability curves and we will get a definite relation between them and the change in length curve as I have shown. Cotton and Mouton[5] suggest that the phenomena of electric and magnetic double refraction may be explained by means of the orientation of particles.

At present I am studying the permeability and induction of a group of oblate spheroids of soft iron which I can orient mechanically about axes lying in their equatorial planes. Their behavior, I believe, will throw much light on this phase of the subject.

[1] Bidwell, Proc. Roy. Soc., p. 228, Vol. 55, 1894.

[2] Due to faintness of photograph the elongation of Rod 1 for amperes 3.2, Fig. 4, is indicated by arrows.

[3] Vol. 5, Part 2, Elek. u. Mag., pp. 48–49.

[4] Loc. cit.

[5] Cotton and Mouton, Comptes Rendus, t. 147, p. 193, 1908.

SUMMARY.

1. These experiments have developed a very simple method for rapidly securing a series of readings on the Joule magnetostrictive effect, thus eliminating heat effects. The method is not only applicable to this effect but to the Wiedemann magnetostrictive effect by simply rotating the camera through 90° and arranging the lamp filament in a vertical direction. It is also applicable in obtaining permanent records of the magnetometer in magnetometric studies of the magnetic qualities of iron, steel, etc.

2. This study has established certain relations between the induction and change in length which suggest from the standpoint of the orientation theory that the permeability and induction curves are the resultant curves of a group of small elements which also possess the property of permeability, but that the resultant curves of permeability and induction are not the same as that of the elementary particles because they are variously oriented and produce mutual effects which change the character of the resultant curves.

3. The results obtained from rod 1 point to a similarity in the behavior of iron and steel for the Joule effect.

PHYSICAL LABORATORY,
OBERLIN COLLEGE, OBERLIN, OHIO,
December, 1911.

NOTE ON THE OPTICAL EFFECTS IN MOVING MEDIA.

By Y. Laub.

IN the second volume (second edition) of his book Theorie der Elektrizitaet, 1908, Professor Abraham proposes a modification of Einstein's electrodynamical theory. He abandons the principle of a constant velocity of light, and introduces in addition to the relativity principle the assumption of a non-constant velocity, according to which the velocity of light depends on the motion of the observer even in vacuum. If c denote the velocity of light as measured by an observer at rest, the velocity c' as measured by an observer moving with velocity v is given by the equation

$$c' = c\sqrt{1 - \frac{v^2}{c^2}}.$$

Conversely the time of a watch is independent of the movement of the reference system.

The electromagnetic equations have in spite of the non-constant light velocity, an invariant character, since in them we are only concerned with the product of light velocity and time t, so that the equation

$$c't' = ct\sqrt{1 - \frac{v^2}{c^2}}$$

still holds.

Following on my former work,[1] I shall here show that it is possible to differentiate experimentally between the principle of constant and non-constant light velocity.

From the observations of many experimenters it is definitely established that the canal rays give a displaced line spectrum. Let light rays be propagated through a tube where canal rays are being produced, and consider the dispersion of the light. We form the following picture of the process of dispersion. There are present in the moving canal rays resonating couplets. When light is propagated in the tube, the couplets are set in vibration, but the whole particle remains electrically uncharged,

[1] Y. Laub, Sitzungsb. d. Heidelberger Akademie, Math. nat. Kl., 6 Abh., 1909; Ann. d. Physik, Bd. 29, 1909, and Beibl. d. Phys., 1910.

so that neither the direction nor the magnitude of the velocity of the particles suffers any alteration. To obtain the index of refraction of the light we proceed as follows: We introduce two systems of coordinates, K and K'. If matter is present, which is at rest relative to K', we have the following equations for the system K':

$$(1a)\qquad \begin{cases} \dfrac{d^2\mathbf{P}_x'}{dt'^2} = -n_0^2\mathbf{P}_x' - 2k\dfrac{d\mathbf{P}_x'}{dt'} + \sum\dfrac{eN'}{\mu}\mathbf{E}_x', \\ \dfrac{d^2\mathbf{P}_y'}{dt'^2} = -n_0^2\mathbf{P}_y' - 2k\dfrac{d\mathbf{P}_y'}{dt'} + \sum\dfrac{eN'}{\mu}\mathbf{E}'_y, \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot, \end{cases}$$

where the vector $\mathbf{P}'$ is the electric polarization, the vector $\mathbf{E}'$ is the electric force, μ the mass of the electron, e the electric charge, n_0 the frequency of the free vibration, k a constant (coef. of viscosity) of the medium.

We now consider a second rectangular system K. The origin of K' moves with a constant velocity v in the direction of the positive x-axis of K. Then from the relativity principle:

$$(2)\qquad \begin{cases} x' = \beta(x - vt), \\ y' = y, \\ z' = z, \end{cases} \qquad \left(\beta = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}\right)$$

$$(3)\qquad t' = \beta\left(t - \frac{v}{c^2}x\right);$$

where x, y, z and t refer to the system K.

Further we have between the electric and magnetic vectors $\mathbf{E}'$, $\mathbf{D}'$, $\mathbf{H}'$, $\mathbf{B}'$, $\mathbf{P}'$, referred to the system K.[1]

$$(4)\qquad \begin{cases} \mathbf{E}_x' = \mathbf{E}_x, \\ \mathbf{E}_y' = \beta\left(\mathbf{E}_y - \dfrac{v}{c}\mathbf{B}_z\right), \\ \mathbf{E}_z' = \beta\left(\mathbf{E}_z + \dfrac{v}{c}\mathbf{B}_y\right); \\ \mathbf{D}_x' = \mathbf{D}_x, \\ \mathbf{D}_y' = \beta\left(\mathbf{D}_y - \dfrac{v}{c}\mathbf{H}_z\right), \\ \mathbf{D}_z' = \beta\left(\mathbf{D}_z + \dfrac{v}{c}\mathbf{H}_y\right); \end{cases}$$

[1] A. Einstein u. Y. Laub, Ann. d. Phys., 26, p. 534, 1908.

$$(5)\qquad \begin{cases} \mathbf{H}_x' = \mathbf{H}_x, \\ \mathbf{H}_y' = \beta\left(\mathbf{H}_y + \frac{v}{c}\mathbf{D}_z\right), \\ \mathbf{H}_z' = \beta\left(\mathbf{H}_z - \frac{v}{c}\mathbf{D}_y\right); \\ \mathbf{B}_x' = \mathbf{B}_x, \\ \mathbf{B}_y' = \beta\left(\mathbf{B}_y + \frac{v}{c}\mathbf{E}_z\right), \\ \mathbf{B}_z' = \beta\left(\mathbf{B}_z - \frac{v}{c}\mathbf{E}_y\right); \end{cases}$$

$$(6)\qquad \begin{cases} \mathbf{P}_x' = \mathbf{P}_x, \\ \mathbf{P}_y' = \beta\left(\mathbf{P}_y + \frac{v}{c}\mathbf{Q}_z\right), \\ \mathbf{P}_z' = \beta\left(\mathbf{P}_z - \frac{v}{c}\mathbf{Q}_y\right). \end{cases}$$

If we apply the transformations already used in the equations (2–6) to the equations (1*a*) and remember that for the observer moving with the system the magnetic polarization $Q = 0$ we easily obtain the following equations:

$$(1b)\qquad \begin{cases} \beta^2\frac{\partial^2\mathbf{P}_x}{\partial t^2} + \beta^2 v^2\frac{\partial^2\mathbf{P}_x}{\partial x^2} + 2\beta^2 v\frac{\partial^2\mathbf{P}_x}{\partial x\partial t} = -n_0^2\mathbf{P}_x - 2\beta k\frac{\partial\mathbf{P}_x}{\partial t} - 2\beta kv\frac{\partial\mathbf{P}_x}{\partial x} + \beta\rho\mathbf{E}_x, \\ \beta^2\frac{\partial^2\mathbf{P}_y}{\partial t^2} + \beta^2 v^2\frac{\partial^2\mathbf{P}_y}{\partial x^2} + 2\beta^2 v\frac{\partial^2\mathbf{P}_y}{\partial x\partial t} \\ \qquad = -n_0^2\mathbf{P}_y - 2\beta k\frac{\partial\mathbf{P}_y}{\partial t} - 2\beta kv\frac{\partial\mathbf{P}_y}{\partial x} + \beta^2\rho\left(\mathbf{E}_y - \frac{v}{c}\mathbf{B}_z\right), \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \end{cases}$$

$$\left(\rho = \Sigma\frac{\epsilon^2 N}{\mu}\cdot\right)$$

We now consider waves of light propagated at *right* angles to the direction of motion of the canal rays. Integrating the differential equations we obtain for the x component:

$$(7)\qquad \mathbf{P}_x\{n_0^2 - \beta^2 n^2 + 2\beta kin\} = \rho\beta\mathbf{E}_x.$$

Again Maxwell's equations[1] give the result:

$$(8)\qquad \mathbf{P}_x = \mathbf{E}_x(c^2\gamma^2 - 1).$$

[1] A. Einstein u. Y. Laub, l. c.

From (7) and (8) we have:

$$c^2\gamma^2 - 1 = \frac{\beta\rho}{n_0^2 - \beta^2 n^2 + 2\beta k i n}. \tag{9}$$

If we put

$$c\gamma = \nu - i\kappa, \tag{10}$$

then ν is the index of refraction and κ the coefficient of extinction of the gas.

For simplicity we write:

$$a = \rho \frac{n_0^2 - \beta^2 n^2}{(n_0^2 - \beta^2 n^2)^2 + 4\beta^2 k^2 n^2},$$

$$b = \rho \frac{2\beta k n}{(n_0^2 - \beta^2 n^2)^2 + 4\beta^2 k^2 n^2}.$$

Then equating real imaginary parts

$$\nu^2 - \kappa^2 = 1 + a, \tag{11}$$

$$2\nu\kappa = b. \tag{12}$$

If we can neglect the absorption and take $k = 0$, we have:

$$\nu^2 = 1 + \rho \frac{\beta}{n_0^2 - \beta^2 n^2}. \tag{13}$$

If we limit ourselves to magnitudes of the second order in v/c we have for the index of refraction the formula:

$$\nu^2 = 1 + \frac{\rho\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)}{n_0^2 - n^2} - \frac{v^2}{c^2}\frac{\rho n^2}{(n_0^2 - n^2)^2}. \tag{14}$$

In the neighborhood of the free period n_0 of the gas we can put $n = n_0 + d$, where d is very small. Then the formula (14) becomes

$$\nu^2 - 1 = \frac{\rho}{2nd} - \frac{v^2}{c^2}\frac{\rho}{4d^2}. \tag{15}$$

As we see it follows from Einstein's theory that the index of refraction depends on the velocity in the manner indicated in equation (15) when the velocity of the carriers in the canal rays is at *right* angles to the direction of observation. *Analogous effects hold for the absorption.*

If we put $\mathbf{E}_x = 0$ in the equation (1) we get an equation whose meaning we can express as follows:

Let n_0 the frequency of the light when the source is at rest.

If the source of light moves with a velocity v perpendicular to the

direction of observation, then the frequency n as measured by an observer at rest is given by the equation:

$$n = \frac{n_0}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{16}$$

We may call *this the transverse Doppler effect.*

Let us proceed from the standpoint of a *non-constant* velocity of light. Then according to Abraham, the Lorentz-Einstein equations of transformation for space and time coordinates must be written:

$$x' = \beta\left(x - \frac{v}{c}l\right),$$

$$y' = y,$$

$$z' = z;$$

$$l' = \beta\left(l - \frac{v}{c}x\right),$$

where l is the path traversed by the light. On the other hand the equations for the electric and magnetic vectors remain unaltered, as in equations (4)–(6).

Let us put the variable

$$l' = c't'$$

in the differential equations. Proceeding as above and remembering that $c' = c\sqrt{1 - v^2/c^2}$ we finally obtain in place of (16):

$$n = n_0. \tag{17}$$

An experimental decision between the two equations (16) and (17) would *be of special importance, for a confirmation of equation (16) would be direct proof of the relativity of time. A transverse Doppler effect follows from the relativity principle only if the time of a watch is dependent on the state of motion of the observer.*

Further we could decide, whether the velocity of light in vacuo is a universal constant. It may be here observed that recently it has been questioned whether the velocity of light is truly a universal constant.

Assuming as is now possible, that we have canal rays of a velocity the order 2×10^8 cm. per sec., calculation shows that we could expect as transverse Doppler effect a displacement of the D-lines of some hundreds Angström units.

For dispersion measurements it would be best to use canal rays in vapors.

In conclusion let us consider the direct longitudinal Zeeman effect from both standpoints. In the system K', the following equations hold:

$$(18)\qquad \begin{cases} \dfrac{d^2\mathbf{P}_x'}{dt'^2} + \eta H' \dfrac{d\mathbf{P}_y'}{dt'} + n_0^2 \mathbf{P}_x' = 0, \\ \dfrac{d^2\mathbf{P}_y'}{dt'^2} - \eta H' \dfrac{d\mathbf{P}_x'}{dt'} + n_0^2 \mathbf{P}_y' = 0, \\ \dfrac{d^2\mathbf{P}_z'}{dt'^2} + n_0^2 \mathbf{P}_z' = 0, \end{cases}$$

where H' is the extremal magnetic field and $\eta = \dfrac{|l|}{c\mu}$.

If we transform the equations to the system K, we obtain

$$\beta^2 \frac{\partial^2 \mathbf{P}_x}{\partial t^2} + \beta^2 v^2 \frac{\partial^2 \mathbf{P}_x}{\partial x^2} + 2\beta^2 v \frac{\partial^2 \mathbf{P}_x}{\partial x \partial t} + \beta^3 \eta \left(H - \frac{v}{c}\Theta \right) \frac{\partial \left(\mathbf{P}_y + \frac{v}{c}\mathbf{Q}_z \right)}{\partial t}$$

$$+ \beta^3 \eta v \left(H - \frac{v}{c}\Theta \right) \frac{\partial \left(\mathbf{P}_y + \frac{v}{c}\mathbf{Q}_z \right)}{\partial x} + n_0^2 \mathbf{P}_x = 0,$$

$$\beta^2 \partial^2 \left(\mathbf{P}_y + \frac{v}{c}\mathbf{Q}_z \right) + \beta^2 v^2 \frac{\partial^2 \left(\mathbf{P}_y + \frac{v}{c}\mathbf{Q}_z \right)}{\partial x^2} + 2\beta^2 v^2 \frac{\partial^2 \left(\mathbf{P}_y + \frac{v}{c}\mathbf{Q}_z \right)}{\partial x \partial t}$$

$$- \beta \eta \left(H - \frac{v}{c}\Theta \right) \frac{\partial \mathbf{P}_x}{\partial t} - \beta \eta v \left(H - \frac{v}{c}\Theta \right) \frac{\partial \mathbf{P}_x}{\partial x} + n_0^2 \left(\mathbf{P}_y + \frac{v}{c}\mathbf{Q}_z \right) = 0,$$

$$\beta^2 \frac{\partial^2 \left(\mathbf{P}_z - \frac{v}{c}\mathbf{Q}_y \right)}{\partial t^2} + \beta^2 v^2 \frac{\partial^2 \left(\mathbf{P}_z - \frac{v}{c}\mathbf{Q}_y \right)}{\partial x^2} + 2\beta^2 v \frac{\partial^2 \left(\mathbf{P}_z - \frac{v}{c}\mathbf{Q}_y \right)}{\partial x \partial t}$$

$$+ n_0^2 \left(\mathbf{P}_z - \frac{v}{c}\mathbf{Q}_y \right) = 0,$$

where Θ is the "extremal electric displacement." Remembering that for an observer moving with the magnetic field the *electric displacement is zero*, we obtain according to Einstein the following equations:

$$(18a)\qquad \begin{cases} \beta^2 \dfrac{\partial^2 \mathbf{P}_x}{\partial t^2} + \beta^2 v^2 \dfrac{\partial^2 \mathbf{P}_x}{\partial x^2} + 2\beta^2 v \dfrac{\partial^2 \mathbf{P}_x}{\partial x \partial t} + \dfrac{1}{\beta} \eta H \dfrac{\partial \mathbf{P}_y}{\partial t} + \dfrac{1}{\beta} \eta v H \dfrac{\partial \mathbf{P}_y}{\partial x} + n_0^2 \mathbf{P}_x = 0, \\ \beta^2 \dfrac{\partial^2 \mathbf{P}_y}{\partial t^2} + \beta^2 v^2 \dfrac{\partial^2 \mathbf{P}_y}{\partial x^2} + 2\beta^2 v \dfrac{\partial^2 \mathbf{P}_y}{\partial x \partial t} - \beta \eta H \dfrac{\partial \mathbf{P}_x}{\partial t} - \beta \eta v H \dfrac{\partial \mathbf{P}_x}{\partial x} + n_0^2 \mathbf{P}_y = 0, \\ \beta^2 \dfrac{\partial^2 \mathbf{P}_z}{\partial t^2} + \beta^2 v^2 \dfrac{\partial^2 \mathbf{P}_z}{\partial x^2} + 2\beta^2 v \dfrac{\partial^2 \mathbf{P}_z}{\partial x \partial t} + n_0^2 \mathbf{P}_z = 0. \end{cases}$$

If we solve the differential equations (18*a*), we obtain, on Einstein's theory for the vibration frequency of the resolved spectral lines:

$$n_2 - n_1 = \eta H \left(1 - \frac{v^2}{c^2}\right), \tag{19}$$

According to Abraham's theory we get

$$n_2 - n_1 = \eta H \sqrt{1 - \frac{v^2}{c^2}}, \tag{20}$$

when n_1 the frequency of the slower, n_2 of the quicker vibrations.

If we limit ourselves to magnitudes of the second order in v/c we have according to Einstein:

$$n_2 - n_1 = \eta H \left(1 - \frac{v^2}{c^2}\right),$$

and according to Abraham

$$n_2 - n_1 = \eta H \left(1 - \frac{1}{2}\frac{v^2}{c^2}\right).$$

Then it seems that the longitudinal Zeeman effect also affords a possibility of deciding experimentally concerning the relativity of time. The optical phenomena here considered are especially noteworthy, as these not only give information on the relativity principle in the more restricted sense, but also enable us to decide whether the velocity of light is a universal constant.

La Plata,
Department of Geophysics of the National University,
August, 1911.

A DETERMINATION OF THE RATIO OF THE SPECIFIC HEATS AND THE SPECIFIC HEAT AT CONSTANT PRESSURE OF AIR AND CARBON DIOXIDE.

By H. W. Moody.

I. Introduction.

As a result of some work done by Millikan, Chapman, and Moody (1) in investigating the temperature changes in a Wilson fog chamber and a later extension by Chapman (2) of the method to the observation of the temperatures produced, upon expansion, in chambers filled with saturated vapors, it became evident that the thermocouple was exceedingly well adapted to the measurement of instantaneous temperatures. The smallness of the wires now obtainable and, hence, the smallness of the heat capacities and heat conductivities of the leads give it decided advantages over other schemes for the direct measurement of the temperature changes produced by adiabatic expansions.

A marked advance in the accuracy of the determination of γ was made by Lummer and Pringsheim (3) when they first proposed to make this determination depend upon a direct temperature measurement. In their work, they made use of a platinum bolometer strip. Later, Makower (4) experimented according to the same method, save that he employed a platinum thermometer for the measurement of the temperature changes. In view of the fact that there is so little agreement in the values of γ found by earlier observers, it has seemed worth while to make a new determination, using the thermocouple for the detection of the changes in temperature.

II. Experimental Work.

Method of Experiment.—The method used rests directly upon the law of adiabatic expansion of the gas considered. If p and θ are taken as variables, we have, for the case of a perfect gas expanding adiabatically, from an initial pressure, p_1, to a final pressure, p_2,

$$\left(\frac{\theta_1}{\theta_2}\right)^{\gamma} = \left(\frac{p_1}{p_2}\right)^{\gamma-1}, \tag{1}$$

wherein θ_1 and θ_2 are the absolute temperatures before and after expansion and γ is the ratio of the specific heats by constant pressure and by constant volume. Upon the solution of equation (1) for γ, we obtain

$$\gamma = \frac{\log \frac{p_1}{p_2}}{\log \frac{p_1}{p_2} - \log \frac{\theta_1}{\theta_2}}. \tag{2}$$

Hence, to compute γ, we need to be able to measure p_1, p_2, θ_1, and θ_2. The first three of these quantities present little difficulty, since each can be measured with sufficient leisure to permit of careful measurement. To get θ_2, we need at our command such a device as will give us the instantaneous value of the temperature assumed by the gas in the adiabatic expansion, since this temperature changes rapidly from heat flowing in from the walls of the surrounding vessel.

Arrangement of Apparatus.—The arrangement of the apparatus and its operation are shown in Fig. 1. To hold the quantity of gas to be experimented upon, a large glass carboy, C, of about 60 liters capacity was employed. In this were three openings, one of which, O_1, communicated, by the way of the drying system, D, and the manometers, M_1, M_2, with the compression pump or other source of gas. For air, the drying system consisted of a number of bottles of sulphuric acid in series with several long tubes filled with granular calcium chloride. For carbon dioxide, the calcium chloride was replaced by tubes of phosphorus pentoxide, which was renewed whenever it showed any signs of moisture.

The second opening, O_2, was used for the introduction of the thermocouple into the carboy. The thermocouple, made of 1 mil copper and constantan wire, formed a part of one circuit in a potentiometer system. These wires, in order that the couple, c_1, might be placed approximately at the center of the body of gas, were led in through glass tubes. These latter were bent, just inside the walls of the vessel, so that they were spread, leaving a length of 20 or 25 cm. of bare wire with one junction in the middle. The inner end of these tubes were sealed with heavy wax in order to shut off communication with the outside. The second junction of the couple was enclosed in a small capsule, c_2, just outside the carboy, and so was always at the temperature of the bath, B, in which the carboy itself was placed. The lead wires continued from these capsules through glass tubes to the surface of the water bath, thence to the potentiometer system.

The other opening, O_3, was the avenue of outflow for the gas at expansion. It was closed by a large valve which, when opened, formed a vent of 8 sq. cm. and so made possible a very sudden expansion since this valve could be operated rapidly.

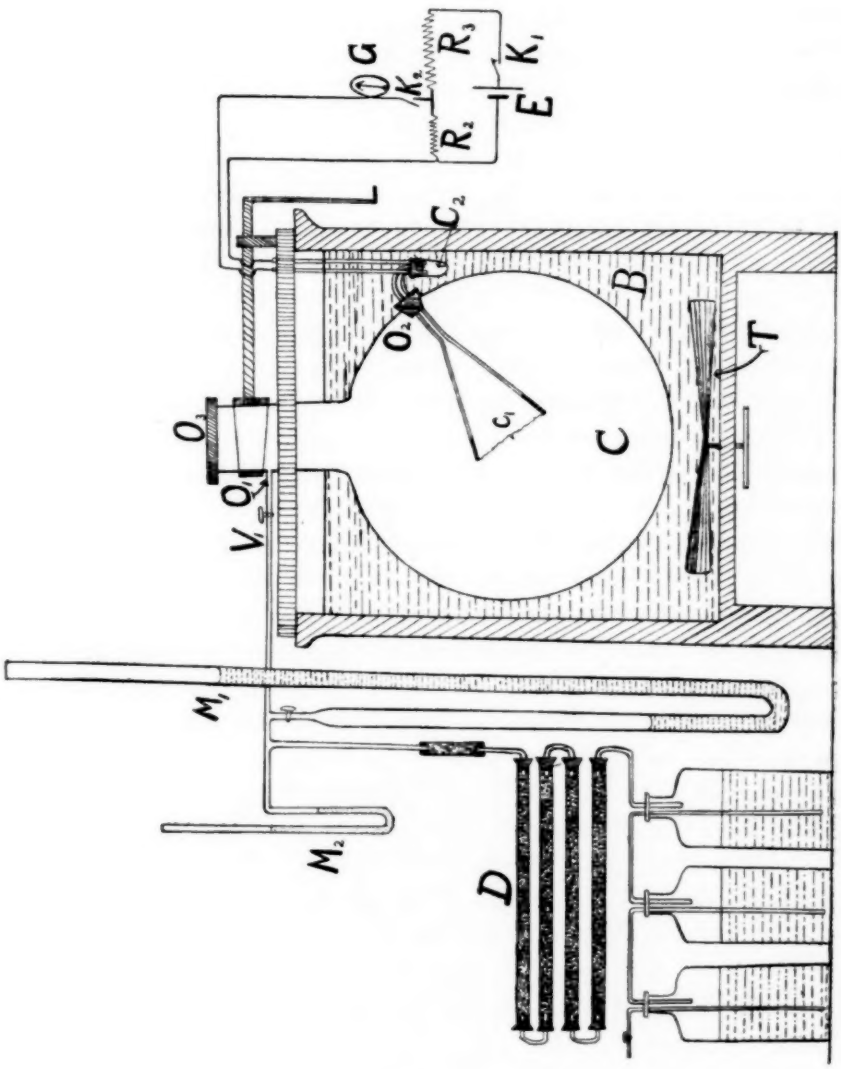


Fig. 1.

The carboy was supported in the center of a large water bath. The stirring device consisted in a large turbine fan, T, motor driven and placed in the center of the bottom of the tank. That the circulation of the water was excellent was evidenced by the movement of particles in the

water. To test the uniformity of temperature throughout the tank, a sensitive thermometer was carried about, readings being taken at many points, and in no place was a variation noticed in excess of a few thousandths of a degree, an amount which could be accounted for by the effect of the variation of pressure on the thermometer bulb. That the temperature of the bath might be controlled, it was equipped with a mercury thermostat, not shown in the figure; the mercury tube being 2.5 cm. in diameter and 150 cm. in length and placed so that it extended from top to bottom and across the bottom of the tank. The thermostat system consisted in the mercury tube, in which the mercury, rising with the rise in temperature of the bath, bridged a circuit, I., through a battery, or other source of current, and an electromagnet. This latter, in turn, lifted an iron core to which was attached a key in circuit II., containing a set of incandescent lamps. Thus was comprised a system by means of which the temperature of the bath could be maintained constant to .01° for any length of time.

Measurement of Pressures.—In the measurement of pressure, the final pressure, p_2, was that of the atmosphere, registered upon a Schneider standard mercury barometer. This instrument had but recently been compared with the standard barometer of the U. S. Weather Bureau service, kept in the Federal Building at Chicago. During the experiments, it was kept in the room, near the manometers, and its reading taken immediately after each observation. Since this reading could be made with an accuracy of .1 mm. mercury, its influence could not introduce an error of more than .01 of 1 per cent. in the final result.

The initial pressure, p_1, was equal to that of the barometer reading plus an excess pressure which was registered upon a " transformer " oil manometer, M_1, and, as a check, upon a mercury manometer, M_2. The readings upon this oil manometer were made by means of a reading glass or microscope, held perpendicularly to the tube. Since the manometer had an internal diameter of 1 cm., a fair portion of the surface was horizontal so that by the above scheme the excess pressure could be measured to .1 mm. of oil. This made the possible error in p_1, expressed in mercury, not more than .06 of 1 per cent. at the lower pressures and .03 of 1 per cent. at the higher pressures.

For the conversion of the manometer readings in oil into terms of mercury, a temperature-density curve for the oil was plotted over a range of 10°, from 16° C. to 26° C. The mercury temperature-density curve was plotted from data taken from Castell-Evans Physico-Chemical Tables (5).

The valve, V_1, was closed immediately before an expansion and the reading taken immediately after, if the observation was satisfactory. Then the adjusting of pressures for a new trial was done, approximately, by the aid of a mercury manometer, M_2, so that the oil was not allowed to move up and down the tube for more than a few cms. This avoided errors in subsequent readings due to the gradual running down of oil that had adhered to the walls of the manometer tube.

Measurement of Temperature.—The temperature, θ_1, was that of the constant temperature bath in which the carboy was submerged. This was observed by the reading of a Baudin standard mercury thermometer. The readings upon the standard were made by the aid of a microscope, having a comparison scale in the eye piece. In this way, θ_1 was obtainable to .01° C., with interpolation to .001° possible. The error in γ introduced thus was not more than .01 to 1 per cent.

To obtain θ_2, the procedure is as follows: The thermal electromotive force developed in the couple circuit by a change in temperature incident to an expansion was evaluated in terms of the E.M.F. of a storage cell by means of a potentiometer made up from resistance boxes. The boxes had been standardized at the Bureau of Standards, Washington, D. C., and afforded a total of 120,000 ohms in the battery circuit and read by .5 ohm up, in the couple circuit, so that the thermal E.M.F. was known in microvolts. Immediately after each observation, the E.M.F. of the storage cell was evaluated by comparison with a Weston standard cell, which had a certificate from the Physikalische Technische Reichsanstalt.

Having, now, the value of the fall in temperature in terms of the thermal E.M.F., it remained to determine, by direct observation, the difference of temperature necessary to develop the same E.M.F. This was done by removing the couple from the carboy and placing the constant temperature junction in a bath at 25° C. and the second junction in another bath at a lower temperature. By this means, a temperature E.M.F. curve was plotted. For succeeding observations, it was only necessary to measure the thermal E.M.F. and, at once, θ_2 could be read from this calibration curve. This curve was plotted to such a scale that θ_2 could be read directly to .01° C. and, by interpolation, to .001° C. The error introduced in γ, thus, was not more than .01 of 1 per cent.

Systematic Errors.—Since some heat inflow will occur, the conditions of the experiment do not furnish a perfectly adiabatic expansion. The heat brought in by gaseous conduction is certainly negligible. That due to convection is small since the temperature is uniform on all sides and, so, there is little cause for circulation, save for that which is due to the

surging and swirls arising at the sides of the outlet valve. Of these more will be said. Since we are dealing with wires of only .000009 sq. cm. cross section, it is seen that the heat capacity is very small and, likewise, the amount of heat which would actually be brought in by metallic conduction.

A greater source of error lies in the fact of internal radiation. The amount of this is a function of the emission coefficient of the glass and of the absorption coefficients of the copper and constantan wires. An experimental attempt at evaluating this was made. A determination of γ was made for a certain pressure with a bright couple and, then, again, after the couple had been blackened with a thin coat of "optical black." In the latter case, a smaller value of γ was obtained, owing to a higher coefficient of absorption and, hence, an apparently less fall in temperature registered. Then other couples were made, one bright and one blackened, and the two were placed in a vacuum tube. The ratio of the absorption coefficients was determined by comparing the times required for each, in turn, to acquire the same temperature when exposed to the same source of radiant energy. This ratio, as a mean of 12 observations, ranging from 9.2 to 10.3, was found to be 10. From this, it follows that we will make a close approximation to the correct value of γ, if we add to the value obtained with the bright couple one ninth of the difference between the values obtained with the bright and blackened couples. That this correction might not be a function of a particular couple, the difference was obtained on several couples, as shown in the following table:

TABLE I.

Pressure.	Bright Couple.	Blackened Couple.	Difference.
12.540	1.3980	1.3800	.0180
43.531	1.3932	1.3757	.0175
12.501	1.3962	1.3775	.0187
43.60	1.3926	1.3748	.0178
		Mean difference.....	.0180

Hence the radiation correction is + .0020.

It was found advantageous, in order to avoid errors due to swirling eddies in the gas, following an expansion, to close the potentiometer circuit and to open the galvanometer damping key after an interval of two or three seconds. During this time, more or less heat had flowed in, according to the fall in temperature. To eliminate this, γ was measured at different levels of excess pressures and a pressure-γ curve plotted.

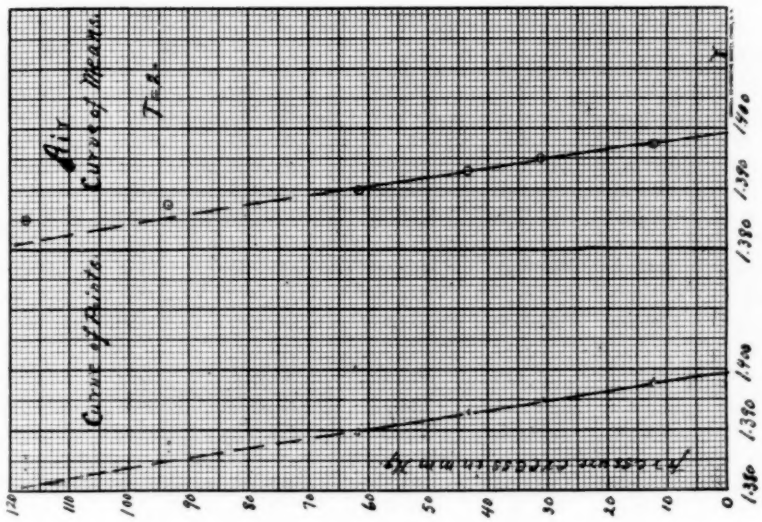

Air
Curve of Means
T. 2.
Curve of Points
Pressure reduced to mm Hg
120
110
100
90
80
70
60
50
40
30
20
10
0
1.350
1.360
1.370
1.380
1.390
1.400
γ

Fig. 3.

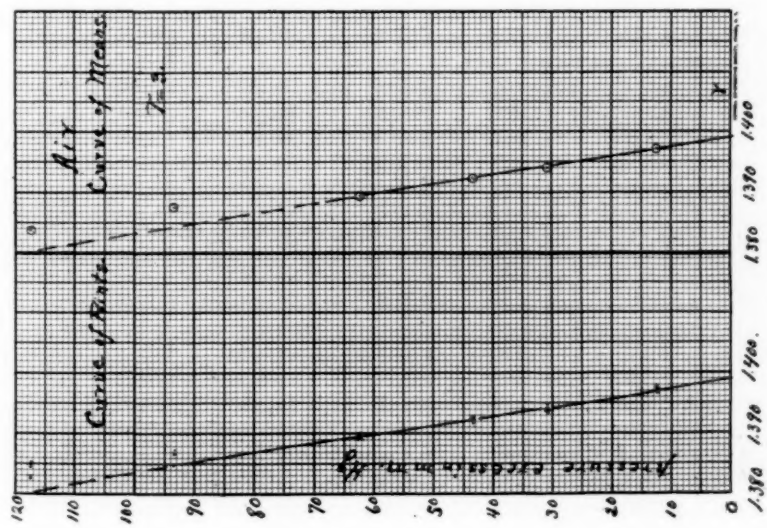

Air
Curve of Means
T. 2.
Curve of Points
Pressure reduced to mm Hg
120
110
100
90
80
70
60
50
40
30
20
10
0
1.350
1.360
1.370
1.380
1.390
1.400
γ

Fig. 2.

TABLE II.

Data on γ for Air, for τ = 3.

p_2	dp	p_1	θ_1	θ_2	γ	D
751.228	12.470	763.698	298.006	296.614	1.3966	−.0006
748.113	12.375	760.448	298.007	296.620	1.3973	+.0001
742.005	12.323	754.328	298.007	296.617	1.3965	−.0007
748.365	12.489	760.834	298.007	296.607	1.3976	+.0004
753.121	12.560	765.681	298.006	296.608	1.3972	.0000
741.027	12.493	753.520	298.008	296.593	1.3980	+.0008
753.058	12.322	765.386	298.007	296.633	1.3972	.0000
746.795	30.868	777.663	298.007	294.617	1.3936	−.0003
750.293	31.032	781.325	298.007	294.612	1.3941	+.0002
749.113	30.893	780.006	298.007	294.617	1.3949	+.0010
753.121	30.996	784.117	298.006	294.626	1.3943	+.0004
754.227	30.942	785.219	298.006	294.646	1.3932	−.0007
740.648	31.002	771.650	298.008	294.575	1.3938	−.0001
754.503	30.836	785.339	298.007	294.652	1.3940	+.0001
744.515	43.520	788.035	298.037	293.307	1.3919	−.0005
751.230	43.436	794.666	298.006	293.316	1.3929	+.0005
745.475	43.372	788.847	298.017	293.317	1.3917	−.0007
750.070	43.500	793.570	298.007	293.305	1.3929	+.0005
734.291	43.501	777.792	298.009	296.214	1.3924	.0000
750.953	43.469	794.422	298.007	293.317	1.3925	+.0001
753.122	43.341	796.463	298.006	293.341	1.3927	+.0003
740.828	43.434	786.262	298.008	293.270	1.3916	−.0008
755.138	43.277	798.415	298.007	293.357	1.3931	+.0007
748.019	62.147	810.166	298.007	291.407	1.3900	+.0005
751.152	62.097	813.249	298.006	291.448	1.3891	−.0004
749.857	62.110	811.967	298.007	291.427	1.3899	+.0004
752.001	62.086	814.087	298.007	291.452	1.3897	+.0002
753.122	61.999	815.121	298.006	291.476	1.3890	−.0005
753.883	62.062	815.945	298.006	291.471	1.3894	−.0001
740.963	62.064	803.027	298.007	291.357	1.3894	−.0001
740.864	62.066	802.930	298.008	291.363	1.3894	−.0001
755.437	62.113	817.433	298.007	291.487	1.3897	+.0002
745.597	93.339	838.936	298.002	288.354	1.3870	+.0004
749.435	93.377	842.812	298.007	288.407	1.3866	.0000
752.001	93.361	845.362	298.007	288.467	1.3849	−.0017
753.221	93.246	846.467	298.006	288.461	1.3868	+.0002
755.763	93.240	849.003	298.007	288.477	1.3876	+.0010
748.679	117.494	866.173	298.017	286.197	1.3843	+.0006
748.350	117.597	865.947	298.007	286.217	1.3823	−.0014
750.307	117.539	867.846	298.007	286.267	1.3822	−.0015
740.704	117.496	858.200	298.008	286.058	1.3849	+.0012
755.813	117.359	873.172	298.007	286.287	1.3852	+.0015

TABLE III.

Data on γ for Air, for $\tau = 2$.

p_2	dp	p_1	θ_1	θ_2	γ	D
751.290	12.430	763.720	298.006	296.616	1.3980	+.0002
748.760	12.440	761.200	298.006	296.614	1.3971	−.0007
753.058	12.322	765.380	298.007	296.632	1.3984	+.0006
751.290	30.981	782.271	298.006	294.616	1.3949	−.0001
748.660	30.942	779.602	298.007	294.607	1.3952	+.0002
754.503	30.836	783.339	298.007	294.647	1.3949	−.0001
751.455	43.551	795.006	298.006	293.306	1.3930	.0000
747.464	43.298	790.762	298.007	293.312	1.3927	−.0003
755.138	43.264	798.402	298.007	293.357	1.3933	+.0003
751.790	62.157	813.947	298.006	291.446	1.3896	−.0001
747.164	61.887	808.051	298.007	291.437	1.3891	−.0006
755.437	61.871	817.308	298.007	291.487	1.3908	+.0009
752.387	93.222	845.609	298.006	288.476	1.3882	+.0007
747.153	93.269	840.422	298.007	288.367	1.3865	−.0010
755.813	93.187	849.000	298.007	288.477	1.3880	+.0005
752.622	117.470	870.092	298.006	286.256	1.3838	−.0013
747.129	117.375	864.504	298.007	286.137	1.3860	+.0009
755.813	117.359	873.172	298.007	286.272	1.3856	+.0004

Having measured γ for excess pressures varying from 12 cm. mercury down to 12 mm., a smooth curve, drawn through the plotted points and extended back to the γ axis, gave the value of γ for $dp = 0$, *i. e.*, when the effect on γ, due to the heat inflow was nil.

Results.—In Table II. is given the data on γ, for the case where the interval between the time of expansion and the time of closing the potentiometer circuit, τ, was 3 sec. These are the observations of several complete series, from high to low pressures, arranged in groups, according to the pressure levels. In these tables, D means the variation of that observation from the mean value for that pressure level.

In Fig. 2 are the graphs of these data, where dp is taken as the ordinate and γ plotted as abscissa. Since the variation in γ is far more than that due to the slight differences in pressure level, it was thought worth while, as a check, to plot a curve of means, *i. e.*, for each level, the mean pressure, for that level, was plotted against the mean γ for that level.

Table III. contains, similarly, the data on γ for air, for the trials in which $\tau = 2$. In Fig. 3 are the corresponding graphs.

In Table IV. will be found the data on γ for carbon dioxide, for $\tau = 3$.

TABLE IV.

Data on γ for Carbon Dioxide, for $\tau = 3$.

p_2	dp	p_1	θ_1	θ_2	γ	D
742.849	12.824	755.673	297.904	296.739	1.2968	+.0002
746.693	12.753	759.446	297.884	296.738	1.2961	−.0005
745.832	12.723	758.555	298.014	296.863	1.2966	.0000
750.213	12.606	762.819	298.034	296.899	1.2967	+.0001
743.674	12.612	756.286	297.944	296.799	1.2969	+.0003
751.052	12.257	763.309	298.004	296.902	1.2967	+.0001
747.103	12.200	759.303	298.004	296.902	1.2965	−.0001
743.299	31.333	774.632	298.074	295.294	1.2934	+.0006
747.172	31.212	778.384	298.014	295.254	1.2942	+.0014
745.398	31.266	776.664	298.014	295.254	1.2927	−.0001
750.858	31.027	781.885	298.014	295.314	1.2913	−.0015
743.574	31.035	774.609	298.014	295.254	1.2931	+.0003
751.253	30.812	782.065	298.004	295.304	1.2926	−.0002
747.103	30.575	777.678	298.004	295.309	1.2928	.0000
743.349	43.692	787.041	298.064	294.259	1.2902	−.0005
747.452	43.642	791.094	298.004	294.219	1.2907	.0000
744.887	43.673	788.560	298.024	294.212	1.2916	+.0009
750.816	43.379	794.195	298.004	294.264	1.2900	−.0007
743.574	43.462	787.036	298.014	294.214	1.2918	+.0011
751.236	43.200	794.436	298.004	294.274	1.2907	.0000
747.191	43.103	790.294	298.004	294.274	1.2899	−.0008
743.374	62.005	805.379	298.014	292.674	1.2886	+.0010
750.836	61.832	812.668	298.004	292.779	1.2878	+.0002
747.079	61.803	808.882	298.004	292.779	1.2868	−.0008
743.349	62.282	805.631	298.064	292.759	1.2873	−.0003
746.920	62.256	809.902	298.004	292.714	1.2881	+.0005
744.558	62.285	806.843	298.044	292.744	1.2875	−.0001
750.816	62.056	812.872	298.004	292.794	1.2873	−.0003
743.349	93.503	836.852	298.064	290.329	1.2852	−.0001
746.741	93.138	839.879	298.014	290.319	1.2862	+.0009
744.558	93.464	838.022	298.044	290.319	1.2854	+.0003
750.134	93.281	843.415	298.004	290.374	1.2841	−.0012
743.062	93.224	836.286	298.014	290.314	1.2850	−.0003
750.836	93.027	843.863	298.004	290.370	1.2855	+.0002
746.967	93.126	840.093	298.004	290.314	1.2862	+.0009
743.049	117.482	860.531	298.054	288.554	1.2831	+.0002
744.628	117.376	862.004	298.014	288.549	1.2842	+.0013
744.567	117.486	862.053	298.044	288.544	1.2838	+.0009
750.134	117.327	867.461	298.004	288.629	1.2820	−.0009
751.048	117.228	868.276	298.004	288.604	1.2826	−.0003
746.967	117.208	864.175	298.004	288.599	1.2820	−.0009

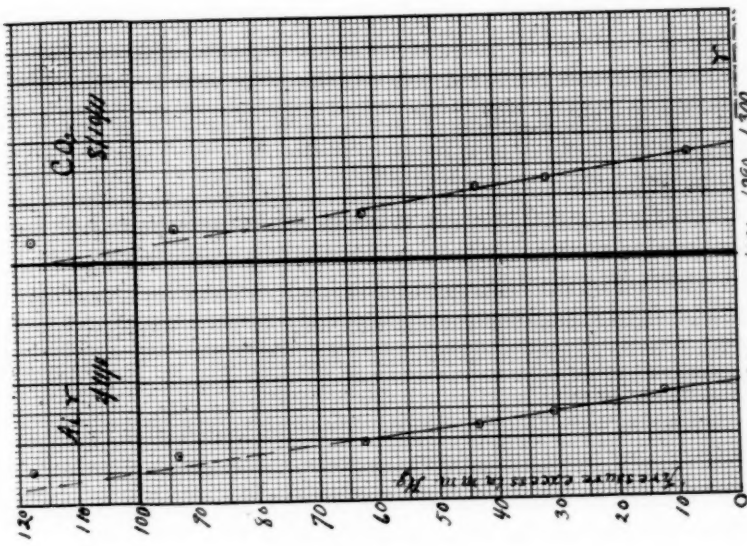

CO2
Slight
Air
High
Excess over 760 mm Hg
T

Fig. 5.

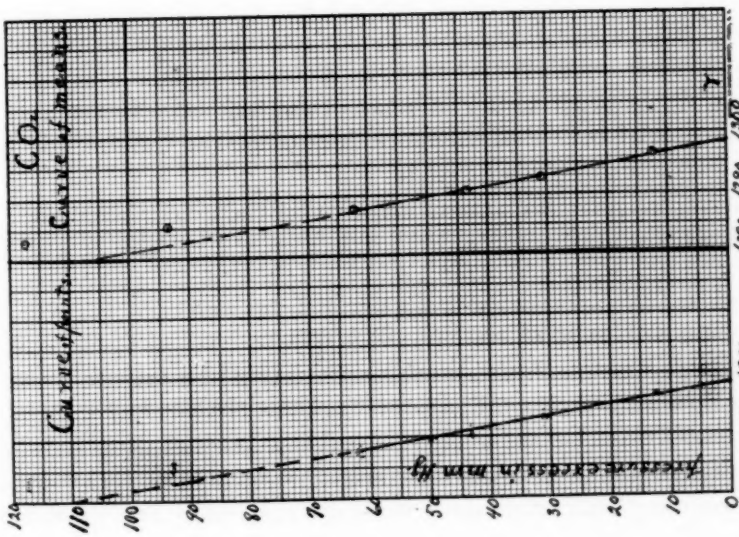

CO2
Curve of means.
Excess over 760 mm Hg
T

Fig. 4.

This was the only interval used in the case of carbon dioxide. The notation is the same as for the tables on air. Fig. 4 contains the graphs for these data.

Thinking it would be of interest to some, we have inserted Fig. 5 to show the degree of consistency which was found in most all cases. These curves are the graphs of single complete series, all points of which were determined within three of four hours of each other, so that all conditions could be maintained practically constant throughout the series.

The mean of all determinations, made in the manner described above, gives for air, $\gamma = 1.3991$, and for carbon dioxide, $\gamma = 1.2983$. Whence, upon the addition of the radiation correction, we have for air, at 25° C., 76 cm., $\gamma = 1.4011$ and for carbon dioxide, at the same temperature and pressure, $\gamma = 1.3003$, in which the probable error is $\pm .0005$.

III. A Review of Recent Determinations of γ for Air and Carbon Dioxide.

In order to compare these values with those found by earlier observers and to draw conclusions as to the probable value of γ, for air and carbon dioxide, we may refer to the summary of A. Winkleman (6) and the critical review of M. G. Maneuvrier (7), both of which appeared about 1895. The results there given show such a wide range of variation in values that we can draw no safe conclusion. It is but natural, however, to expect that, with the improved methods and apparatus of the modern laboratory, the more recent determinations of γ should show a closer agreement. In order to see to what extent this expectation has been realized, a brief review of the work done since the time of the above mentioned summaries will be made.

Indirect Methods.—All indirect methods for the determination of γ are based upon the application of the formula for the velocity of sound,

$$V = n\lambda = \sqrt{\frac{p\gamma}{\rho}}. \tag{3}$$

(*a*) *Long Distance Measurements in Free Air.*—To this method there are the following serious objections: (1) The difficulty of making the proper corrections for the effect of humidity, (2) lack of uniform temperature throughout the path, (3) the relatively high intensity required at the source. This latter must die out as the wave recedes from the source and it is probable that the velocity of the wave diminishes with the intensity. By far the most reliable determination made by this method was that of the French Academy (8), in 1822. They found $V_0 = 331.2$ m. per sec., from which $\gamma = 1.3999$.

(*b*) *Hebb's Method.*—In 1905, Hebb (9), by a very ingenious method, suggested by Professor Michelson, eliminated the objections mentioned above as well as those which apply to the measurements made in tubes, viz., the " personal equation "[1] and the effect of the walls.

His measurements were made in the halls of Ryerson Laboratory. He applied the principle of interference by means of a system of parabolic mirrors, having a tuning fork source placed at the focus of one and a telephone receiver at the focus of the other mirror. He found $V_0 = 331.29$, with a probable error of .06 m. from which $\gamma = 1.4003$.

(*c*) *Measurements in Long and Large Tubes.*—The objections to the measurements in free air have also been eliminated by observers using long and large tubes, though they have introduced a new one, viz., the effect of the walls of the tubes. Violle and Vautier (10) made a first determination at Grenoble, in 1885, in a U-shaped conduit, 70 cm. in diameter and about 13 km. in length. In 1895, they made a second determination in another sewer conduit, just constructed, at Argentuiel; this one being a straight conduit, 3 m. in diameter and 3 km. in length. Very careful corrections were made for temperature, pressure and hygrometric conditions. They found $V_0 = 331.15$ m. per sec. with a probable error of 2 or 3 cm. A final correction for the effect of the walls was made by Violle (39), in 1900, and he gives $V_0 = 331.36$ m. per sec., whence $\gamma = 1.4009$.

(*d*) *Measurements in Short Tubes.*—Many observers have employed the interference method of Quincke with varying success. The difficulties with this method may be most easily pointed out under the discussion of the individual works. (1) J. Webster Low (11) found for air, at 10° C., $V = 330.88$, whence $\gamma = 1.3968$. The influence of the " personal equation " is evident here in the fact that he used maxima of tone strength instead of minima. Later, Stevens has corrected Low's value for humidity and effect of the size of the tubes. He gives as the corrected value, $V_0 = 331.4$, from which we get $\gamma = 1.4012$. (2) O. Buckendahl (12) found certain disturbing resonance phenomena, between the entire tube system and the body of gas, whose existence was not easily discoverable. Then, too, errors often entered because of diffusion with the outside air, so that some question holds against his results. He obtained, for carbon dioxide, at 0° C., 76 cm., $\gamma = 1.2990$. (3) E. H. Stevens (13), using this method and making very careful corrections for humidity, size of tubes, etc., found for air, $V_0 = 331.32$, from which $\gamma = 1.4006$. This seems to be the most reliable determination of all the recent ones according

[1] This expression relates to the error in judgment involved in setting on a maximum or minimum of tone strength.

to this method. (4) R. Fürstenau (14) made some improvements on the method and was able to eliminate diffusion with outside air and the "personal equation" to some extent, still he encountered uncertainties as to the correction for the wave-length because of lack of uniformity in the heating of the tube. He, too, met with certain resonance phenomena in the surrounding tubes. He obtained for carbon dioxide, at 13.5–16.5° C., $\gamma = 1.3008$, and made no measurements on air.

Direct Methods: (*a*) *Assman's Method.*—The mechanical manipulation of this method is simple, but it cannot be considered as a serviceable method because (1) the period (about one half sec.) is so long that the process is not adiabatic and (2) because the volume of the bulbs of gas is too small. The evidence of this is found in the work of (1) Assman (15), himself, who, in 1852, obtained for air, $\gamma = 1.421$ and $\gamma = 1.427$; (2) P. A. Müller (16), who, in 1883, obtained for air, $\gamma = 1.4046$ and for carbon dioxide, at 20° C., $\gamma = 1.2653$; and of B. Hartman (17) who, in 1905, found $\gamma = 1.4137$, as a mean of readings varying from $\gamma = 1.4084$ to $\gamma = 1.4201$. He found for carbon dioxide, $\gamma = 1.3064$, as a mean of values varying from $\gamma = 1.302$ to $\gamma = 1.311$.

(*b*) *The Method of Clement and DesOrmes.*—In this classical method, the determination of γ rests upon the measurement of pressure. The results obtained by experienced observers are exceedingly discordant, varying between 1.302 (Kohlrausch) and 1.42 (Masson). Roentgen's (18) determination has usually been considered the most reliable, but since a number of unsuccessful attempts have been made to duplicate it, some of them in Ryerson Laboratory, under conditions practically identical with those under which Roentgen worked, we are inclined to attach less weight to it than has sometimes been done, and to leave out of consideration all measurements made by the Clement and DesOrmes method.

A slight modification of the above method was made by Maneuvrier and Fournier (19) when they made use of Reech's theorem and they were able to get more consistent results than earlier observers who used the method. Reech's theorem, $\gamma = \partial p_\varrho / \partial p_\theta$, holds only for infinitesimal changes while, in practice, the observer must work with finite changes and then extrapolate back, graphically, to the limiting values. Again, there must be surging in the body of the gas upon expansion—a fact which introduces large uncertainties into the measurement of the final pressure. These, together with the criticisms which apply to any method depending upon a measurement of pressures (see below) make it seem that their values are not entirely reliable. They obtained for air, at room temperature and atmospheric pressure, $\gamma = 1.3924$ and for carbon dioxide, $\gamma = 1.2993$.

(*c*) *Direct Application of the p-θ Adiabatic Equation.*—This is a remarkably simple and direct method, requiring only some device for the direct observation of the final temperature, θ_2, all other quantities being such that they can be measured with the highest degree of accuracy. This method possesses a great advantage over other direct methods in that it makes the determination of γ rest upon a temperature measurement rather than upon a pressure measurement. A pressure measurement is, at best, a measurement of the mean temperature throughout the vessel at the instant of measurement, while the direct temperature measurement can be made upon a small quantity of gas in the very center of the vessel. By the pressure scheme, the temperature observed is always too high because of the inflow of heat which begins at the instant of expansion, whereas the temperature at the center is not affected until a finite time after the expansion. (1) Lummer and Pringsheim (3) were the first to use the method. They used a platinum bolometer strip to indicate the temperature change by the change in its resistance, this latter being measured by means of a Wheatstone bridge. They obtained for air, at 15° C., 76 cm., $\gamma = 1.4025$ and for carbon dioxide, under the same conditions, $\gamma = 1.2995$. (2) W. Makower (4) used a platinum thermometer to detect the temperature change. While this work was very carefully done, the consistency obtained was not all that could be desired. Working under ordinary conditions of temperature and pressure, he obtained for air, $\gamma = 1.401$. (3) The author's work gives for air at 25° C., 76 cm., $\gamma = 1.4011$, and for carbon dioxide, under the same conditions, $\gamma = 1.3003$.

Summary and Tables.—Several attempts at computing the value of γ, from theoretical considerations, have been made, *e. g.*, by Boynton (20), Capstick (21), and Le Duc (22), but in all cases, because of approximations made or uncertainties in the values of certain constants needed, these results do not agree sufficiently to aid us.

In the following table will be found what seem to be the most reliable determinations made during the period covered by this review.

TABLE V.

Table of Observations on γ for Air.

Observer.	Method.	Temperature.	
Violle and Vautier	Velocity of sound.	0° C.	1.4009
Hebb	" " "	"	1.4003
Low (corrected by Stevens)	" " "	"	1.4012
Stevens	" " "	"	1.4006
Lummer and Pringsheim	Application adiabatic equation.	15° C.	1.4025
Makower	" " "		1.401
Moody	" " "	25° C.	1.4011

These values have not been reduced to a common temperature because in the case of air the variation of γ with temperature is inappreciable for the range covered.

TABLE VI.

Table of Observations on γ for Carbon Dioxide.

Observer.	Method.	Temperature.	
Fürstenau.....	Velocity of sound.	13.5:16.5° C.	1.3008
Lummer and Pringsheim.....	Application adiabatic equation.	13° C.	1.2995
Moody.....	" " "	25° C.	1.3003

If we make use of the equation developed by Wüllner (23) and reduce the above values to the same temperature, 20° C., the values of Fürstenau (14) and of Lummer and Pringsheim (3) would each be lessened by about .001, while that of the author would be increased by a like amount. These corrected values give a mean for carbon dioxide, at 20° C., $\gamma = 1.2997$.

IV. A COMPUTATION OF THE SPECIFIC HEAT AT CONSTANT PRESSURE FOR AIR AND CARBON DIOXIDE.

Upon the basis of the first law of thermodynamics, we may deduce the expression

$$C_p = C_v + \frac{1}{J}\left\{\left(\frac{\partial u}{\partial v}\right)_\theta + p\right\}\left(\frac{\partial v}{\partial \theta}\right)_p. \tag{4}$$

(See Buckingham, § 72.) From equation (4), we have, at once,

$$\lambda \equiv \left(\frac{\partial u}{\partial v}\right)_\theta = J C_p \cdot \frac{\gamma - 1}{\gamma} \cdot \left(\frac{\partial \theta}{\partial v}\right)_p - p. \tag{5}$$

This equation gives us a means of determimimg λ, the latent heat of expansion or the internal work of free expansion. The second law furnishes us with the further relation

$$\lambda = \theta\left(\frac{\partial p}{\partial \theta}\right)_v - p. \tag{6}$$

(See Planck, § 153.) In these equations, (5) and (6), we have two independent means of obtaining λ. In (5), the value of λ depends upon the specific heat at constant pressure, the ratio of the specific heats, and the volume coefficient of expansion. In (6), its value is determined by the thermodynamic temperature, obtained from values of the Joule-Thomson effect, and the pressure coefficient of expansion.

Now, if we make use of the defining equations,

$$\alpha_p = \frac{1}{v_0}\left(\frac{\partial v}{\partial \theta}\right)_p \tag{7}$$

and

$$\alpha_v = \frac{1}{p_0}\left(\frac{\partial p}{\partial \theta}\right)_v \tag{8}$$

we can rewrite our equations. From (5) we have

$$\lambda = JC_p \cdot \frac{\gamma - 1}{\gamma} \cdot \frac{1}{v_0 \alpha_p} - p, \tag{9}$$

and from (6) we get

$$\lambda = \theta p_0 \alpha_v - p. \tag{10}$$

When we insert the values, found, experimentally, for the different quantities, *i. e.*, $C_p = .2375$ and $\gamma = 1.401$, equation (9) gives us a relatively large negative value for λ, while equation (10) gives us a small positive value, which is as we know to be true for most real gases. If an error in γ were the cause of this, it would be necessary for $(\gamma - 1)$ to be about 2 per cent. larger, or $\gamma = 1.408$, in the case of air, in order that we obtain for λ a proper positive value. Since such a value is at variance with all recent reliable determinations of γ, and since it is even more probable that the total errors in all the other quantities is not so great, it seems certain that the generally accepted value of C_p, .2375, is much too small. For this reason, we have thought it worth while to compute it.

Upon the elimination of λ from equations (9) and (10) and the solution for C_p, we obtain an expression for C_p in terms of experimentally determined quantities, viz.,

$$C_p = \frac{\theta\gamma}{\gamma - 1} \cdot p_0 \alpha_v \cdot v_0 \alpha_p \tag{11}$$

which gives C_p in cal/(gm. $\times$ deg.). As was pointed out above, this value of C_p is just as accurate as that of γ, since the values of all other factors are known with a very high degree of accuracy.

In these equations, the symbols used are defined as in the following table:

Symbol.	Definition.
C_p,	Specific heat at constant pressure, in cal./(gm. $\times$ deg.).
C_v,	Specific heat at constant volume.
γ,	Ratio of the two specific heats.
θ,	Temperature on the thermodynamic scale.
θ_0,	Temperature of the freezing point of water on thermodynamic scale.
v_0,	Specific volume, in cm./gm. at the temperature θ_0.
p,	Pressure of one atmosphere in dynes.

J, Mechanical equivalent of heat.
α_p, Volume coefficient of expansion, at temperature θ.
α_v, Pressure coefficient of expansion, at temperature θ.

Since the average room temperature is 20° C., and since all the quantities needed are known more accurately at 20° C. than at 0° C., we have computed the value of C_p for 20° C. and one atmosphere's pressure. In making these computations for 20° C., it must be borne in mind that v_0, at 20° C. and one atmosphere's pressure is not equal to v_0 at 0° C. if we, as we may, arbitrarily define

$$\alpha_v = \frac{1}{p_0}\left(\frac{\partial p}{\partial \theta}\right)_v.$$

In this case, we must define

$$\alpha_p = \frac{1}{v_0'} \cdot \left(\frac{\partial v}{\partial \theta}\right)_p,$$

where

$$v_0' = v_0 \frac{\theta_0}{\theta},$$

on the assumption that Boyle's law holds. This $v_0 = v_0$, at 0° C. and one atmosphere.

As to the available data for use in these computations, we have the following:

1. On the thermodynamic temperature of the freezing point of water, or 0° C., Makower and Noble (24), Rose Innes (25), Pellat (26), Chappuis and Harker (27), Buckingham (28), Callendar (29), and Berthelot (30) have given results, all agreeing fairly well. The more reliable of these results are, probably, those of Callendar, Berthelot, and Buckingham, from which we conclude that $\theta_0 = 273.11°$ on the thermodynamic scale, with an accuracy of $\pm .01°$. The corrections for the interval are negligible, so that 20° C. = 293.11° on the thermodynamic scale.

2. On C_p, the results for air cluster about those of Regnault (31), viz., $C_p = .2375$, as a mean value. But all these earlier observations have been taken over relatively long ranges of temperature. More recently, Swann (32) has made a determination, covering a short range, and has obtained a value higher than that of Regnault's by nearly 2 per cent. He obtained the value $C_p = .2417$, at 20° C. Just as this paper is ready for the press, an abstract has appeared in the *Physikalische Zeitschrift* of the work of Scheele and Heuse (33), who find $C_p = .2408$ at 20° C. For carbon dioxide, a better agreement is found, but, for this, γ seems to be well determined so that we have computed C_p. For carbon dioxide, Regnault (31) gives $C_p = .2025$; Lussana (34), $C_p = .2013$; Holborn and Hennings (35), $C_p = .2039$; Swann (32), $C_p = .2020$, at 20° C.

3. Values of γ have been discussed in Part III. (see above).

4. Since $v = 1/\rho_0$, we need ρ_0, the density at 0° C. The density of air at 0° C., 76 cm., and sea-level has been taken from Landolt-Bornstein's tables. From the same authority, we have taken the value of g, at sea-level, as 980.617. For CO_2 we have taken the value of ρ_0 as given by Lord Rayleigh (36).

5. For the value of J, we need only refer to the summary of Ames (37).

6. Upon the expansion coefficients, the splendid work of Chappuis (38) has been generally accepted as the best yet made, so that his values have been used in this work.

In Table VII. are given the quantities needed in equation (11) and their numerical value in the case of the two gases studied. In Table VIII., we have compared the values computed with Regnault's and the more recent determinations.

TABLE VII.

Table of Values Used in the Computation of C_p.

Magnitude.	Air.	Carbon Dioxide.
γ	1.401	1.300
θ_0	273.11	273.11
ρ_0	.00129278	$1.52909 \times .00129278$
J, at 20° C.	4.181×10^7	4.181×10^7
α_p, at 20° C., 76 cm.	.00367812	.00373447
α_v, at 20° C., 76 cm.	.0036722	.0037067

TABLE VIII.

Values of C_p, Computed and Observed.

Gas.	C_p (Computed).	C_p (Observed).
Air	.24118	.2375 Regnault. .2417 Swann. .2408 Scheele and Heuse.
Carbon dioxide	.2008	.2025 Regnault. .2020 Swann. .2013 Lussana.

It will be seen that our computed value of C_p in the case of air agrees remarkably well with the mean of the two most recent determinations. It is to be noted that the equation used in the computations is perfectly rigorous and all quantities involved are capable of measurement with the highest degree of accuracy. The one in whose value there is the largest uncertainty is γ and there can be little question but that its value is known, accurately, to the third decimal place. In the case of carbon

dioxide, our value agrees within the limits of error with the value given by Lussana, though it is slightly smaller than those found by other observers. Hence it seems that the correct value of C_p for carbon dioxide must lie close about the value found by Lussana.

V. CONCLUSIONS.

1. Our work gives for air, at 20° C., 76 cm., $\gamma = 1.4011$, and for carbon dioxide, under the same conditions, $\gamma = 1.3003$, with a probable error in each of $\pm .0005$. These values agree well with the best determinations of recent years, though the value for air is smaller than some of those of earlier date which have previously been accepted.

2. We find for air, at 20° C., 76 cm., that $C_p = .24118$. This value is so reliable that there can be little doubt but that the values accepted, heretofore, are too small. In fact, a striking confirmation of the correctness of our value of γ is had in the fact that the above value of C_p agrees to within .03 of 1 per cent. with the mean of the recent determinations of Swann and of Scheele and Heuse. For carbon dioxide, at 20° C., 76 cm., we find $C_p = .2008$, which agrees within the limits of error with the value of Lussana and is slightly lower than the values given by other reliable determinations.

In conclusion, the author desires to express his appreciation to the staff of the Physics Department for their kindly interest in this work and especially to Professor Millikan, at whose suggestion and under whose direction the investigation was carried out.

RYERSON PHYSICAL LABORATORY,
THE UNIVERSITY OF CHICAGO,
January 2, 1912.

VI. BIBLIOGRAPHY.

1. Millikan, Chapman, and Moody, PHYS. REV., 30: 286, 1910.
2. Chapman, PHYS. REV., 32: 561, 1911.
3. Lummer and Pringsheim, Smithsonian Contrib. to Know., XXIX., No. 1126; Wied. Ann. d. Phys., 64: 555, 1898.
4. Makower, Phil. Mag. (6), 5: 226, 1903.
5. Castell-Evans, Physico-Chem. Tables, p. 218.
6. A. Winkleman, Handbuch d. Phys., 2: 381, 1896.
7. M. G. Maneuvrier, Jour. d. Phys. (3), 4: 341 and (3), 4: 445, 1895.
8. French Academy, Ann. d. Chim. et d. Phys., XX., 210, 1822.
9. Hebb. PHYS. REV., 20: 89, 1905.
10. Violle and Vautier, Ann. d. Chim. et d. Phys., XIX., 306, 1890.
11. J. Webster Low, Phil. Mag. (5), 38: 249, 1894.
12. O. Buckendahl, Diss. Heidelberg, 1906.
13. E. H. Stevens, Ann. d. Phys., 4, 7: 285, 1902.
14. R. Fürstenau, Verh. d. Deutsch. Gesell., 10: 968, 1908.
15. Assman, Pogg. Ann., 85: 1, 1852.
16. P. A. Müller, Wied. Ann., 18: 94, 1883.

17. B. Hartman, Diss. Gottingen, 1905.
18. Roentgen, Pogg. Ann., 148: 580, 1873.
19. Maneuvrier and Fournier, C. R., 123: 228, 1896.
20. Boynton, PHYS. REV., 12: 353, 1901.
21. Capstick, Proc. Roy. Soc., 57: 322, 1895.
22. Le Duc, C. R., 127: 659, 1898.
23. Wüllner, Wied. Ann., 4: 321, 1878.
24. Makower and Noble, Proc. Roy. Soc., 72: 379, 1903.
25. Rose-Innes, Phil. Mag. (6), 2: 130, 1901.
26. Pellat, C. R., 136: 809, 1903.
27. Chappuis and Harker, Trav. du Bur. Int. des Poids et Mes., XII., 1902.
28. Buckingham, Bull. Bur. Standards, 3: 237, 1907.
29. Callendar, Phil. Mag. (6), 5: 48, 1903.
30. Berthelot, Trav. et Mem. du Bur. Int. des Poids et Mes., XIII., 1907.
31. Regnault, Mem. de l'Acad., 26: 1, 1862.
32. Swann, Phil. Trans. Roy. Soc., 210: 199, 1909.
33. Scheele and Heuse, Phys. Zeit., 12, 24: 1674, 1911, and 13, 2: —, 1912.
34. Lussana, Nuovo Cimento, 3, 36: 5, 70, 130, 1894.
35. Holborn and Hennings, Ann. d. Phys., 23: 809, 1897.
36. Lord Rayleigh, Proc. Roy. Soc., 62: 204, 1897.
37. Ames, Congres Int. d. Phys., 1: 178, 1900.
38. Chappuis, Trav. et Mem. du Bur. Int. des Poids et Mes., XIII., 1903.
39. Violle, Congres Int. d. Phys., 1: 228, 1900.

ON THE PASSAGE OF γ RAYS OF RADIUM THROUGH MATTER.

BY S. J. ALLEN.

INTRODUCTION.

DURING the last few years a considerable amount of work has been done on this subject by various experimenters, and if one reads through their papers one is struck with the wide differences in the results. In very few cases can these be due to experimental errors, but must be ascribed to the effect of the so-called secondary rays produced when the γ rays strike upon and emerge from the surrounding objects.

A very good summary and discussion of past work is given in several papers by Soddy and Russell,[1] to which the reader is referred. They have examined very carefully the effect of different experimental dispositions of the apparatus, and also of different kinds of electroscopes. They are of the opinion that the effects of secondary rays can be eliminated by special arrangements, and that when this has been produced the γ rays are homogeneous throughout their entire range. The absorption of γ rays by matter can be roughly divided into two groups; a very irregular initial portion in which λ/d varies widely with the thickness traversed, and a portion where after a considerable thickness has been passed the value of λ/d is more constant. It has generally been explained in the past that these phenomena are due to the fact that the rays are originally heterogeneous and as successive thicknesses are passed the softer rays are absorbed more rapidly than the harder, thus rendering the remaining rays more homogeneous.

Soddy and Russell's view that the rays are homogeneous is questioned in a paper by R. D. Kleeman.[2]

Before one can decide finally it is necessary that a large number of results on many different materials should be at hand, and that these results should agree quite closely among themselves, which is not the case at present. It can be seen from Soddy and Russell's work that λ/d is not a constant for different elements, but increases largely as the high atomic weight elements are reached. It is also evident that a hardening effect can be produced by first passing the rays through certain elements. These results have also been obtained by the author, and a brief abstract

[1] Soddy and Russell, Phil. Mag., 1909–1910.

[2] R. D. Kleeman, Phil. Mag., 1910.

has appeared in the PHYSICAL REVIEW. The present paper gives the results in more detail and also additional ones.

EXPERIMENTAL ARRANGEMENTS.

Both the balanced electrometer, and the electroscope methods were tried, and from preliminary experiments it was decided to use the latter. The general arrangement is shown in Fig. 1. The electroscope B was of brass, cylindrical in form, with sulphur bead insulation and a fine gold leaf, the movement of which was read by a reading elescope with micrometer scale. The author had the same trouble with irregular initial readings as is mentioned by Soddy and Russell, though not to so great an extent. This trouble is probably due to several reasons, chief among which are the leak, and "soaking in" effect over the sulphur bead, and the heating effect of the electric lamp used to illuminate the leaf. By removing the lamp to a considerable distance and reflecting the light into the electroscope by a mirror, and making the sulphur bead as small as possible, this trouble was eliminated to a great extent. The leaf was charged by means of a battery of lead accumulators, and the full potential was kept on the leaf for an hour or two before taking readings. After sufficient time was allowed to elapse the conditions always became steady for consistent readings.

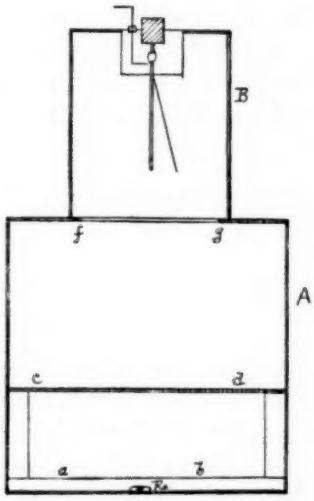


Fig. 1.

The readings could always be repeated to within two tenths of one per cent. The values of λ (the so-called coefficient of absorption) were calculated from the equation,

$$\lambda = \frac{\log_e I_0 - \log_e I_t}{t},$$

where I_0 = the initial intensity of rays, I_t = the intensity after passing through a thickness t of the given material. Wherever possible the successive readings of log I and t were plotted in a curve. In such a case the criterion of the constancy of λ is a straight line. This is the method used by Soddy and Russell. Small variations would however scarcely be distinguished from a straight line, so that it seemed better to the author to test the constancy of λ by successive calculations from the

formula. If the results merely oscillated about a general mean, λ could be considered constant, but if λ varied uniformly in one direction or the other, the rays were not homogeneous. Since the γ rays are very penetrating, unless thick layers of the materials are used the values of I will not differ by much and consequently the calculated value of λ may be considerably in error. For example if two successive readings of I do not differ by more than 10 per cent. and since two tenths per cent. error is possible in each reading the value of λ might be out as much as 2 per cent. For some of the materials the author had not sufficient to form into thick layers, so the errors in these will be greater than others.

The electroscope was placed upon a brass box A, which contained an aluminum platform cd capable of being placed at any desired distance between the base of the electroscope fg and the bottom of the box ab. The radium Ra was placed at the bottom of the box in a small thin glass tube. The rays from this radium practically all came from a point source.

EFFECT OF VARYING THE POSITION OF ABSORBING LAYER, AND THE THICKNESS OF ELECTROSCOPE BASE.

A large number of experiments were made upon the effect of different dispositions of the radium and absorbing layer, and the variation in thickness of the electroscope base. These experiments are general in character, and the results show very clearly how it is possible to obtain such widely divergent values of λ under different experimental conditions.

The results of these are tabulated in Table I. The first column gives the substance used as the absorbing layer, the second the thickness of the layer from which λ was calculated, the third the value of λ, and the fourth the value of λ/density $\times$ 100.

The ratio λ/density can be considered as proportional to the specific absorption per unit of mass for any one material, and will serve as a relative comparison of the absorbing power of different materials according to their atomic weights, since the cone of rays will be approximately the same for each. The true specific absorption would be defined as the stopping power of a cylinder of the material, one square centimeter in cross section, and one gram in mass. If the stopping power is independent of the nature of the atoms of the material then we should expect to find λ/d constant for each material, provided each is tested under the same conditions and with the same kind of ray.

The different experiments will be taken up in detail so as to bring out the distinguishing features.

Experiment 1.—In this experiment the radium was covered with 7

TABLE I.

Substance.	Thickness, Mm.	λ	λ/d × 100
Experiment 1.			
Platinum	0–1.0	1.043	4.85
Mercury	0–4.0	.669	4.92
"	4–12	.589	4.32
"	12–16	.538	3.96
"	16–20	.466	3.43
Bismuth	0–3.25	.515	5.25
Lead	0–3	.621	5.44
"	3–6	.554	4.87
"	6–12	.518	4.54
"	12–15	.509	4.47
"	15–18	.503	4.41
"	18–21	.501	4.39
Palladium	0–1	.471	4.09
Silver	0–3.2	.344	3.27
Copper	0–6	.242	2.72
"	6–9	.273	3.07
"	9–12	.283	3.19
"	12–15	.295	3.31
"	15–18	.302	3.39
Nickel	0–4	.252	2.95
"	4–8	.347	4.07
Wrought iron	0–3	.206	2.64
" "	3–6	.212	2.72
" "	6–9	.235	3.01
" "	9–12	.254	3.25
" "	12–15	.272	3.50
" "	15–18	.291	3.73
" "	18–30.5	.305	3.91
Aluminum	0–8.7	.0736	2.60
Sulphur	0–6.9	.0774	3.81
Experiment 2.			
Bismuth	0–3.25	.464	4.73
Silver	0–3.2	.344	3.27
Copper	0–2.15	.323	3.63
Wrought iron	0–6.1	.245	3.14
" "	6.1–12.2	.246	3.15
" "	12.2–18.3	.243	3.11
" "	18.3–24.3	.249	3.19
Experiment 3.			
Lead	0–3.36	.769	6.75
"	3.36–9.96	.640	5.61
Silver	0–3.2	.508	4.84
Copper	0–2.15	.448	5.03

Substance.	Thickness, Mm.	λ	λ/d × 100
Experiment 4.			
Radium covered with 10.5 mm. lead.			
Lead	0–3.36	.644	5.65
Radium covered with 14 mm. lead.			
Lead	0–3.36	.620	5.44
Radium covered with 21.1 mm. lead.			
Lead	0–3.36	.584	5.12
Radium covered with 31.2 mm. lead.			
	0–3.36	.494	4.33
Radium uncovered.			
Lead	0–3.36	.664	5.82
Experiment 5.			
Mercury	0–4.08	.666	4.89
"	4.08–8.16	.598	4.39
"	8.16–12.24	.564	4.15
Bismuth	0–3.25	.607	6.19
Silver	0–3.2	.442	4.21
Copper	0–3.35	.331	3.72
"	3.35–6.69	.347	3.90
"	6.69–10.0	.328	3.66
"	10.0–13.4	.313	3.52
Nickel	0–4	.357	4.49
Wrought iron	0–6.1	.296	3.79
Cast iron	0–10.1	.259	3.67
" "	10.1–20.1	.247	3.49
Sulphur	0–6.9	.0897	4.46
Aluminum	0–6.83	.110	3.88
Experiment 6.			
Radium covered with 31 mm. lead.			
Silver	0–3.2	.435	4.14
Copper	0–3.36	.281	3.16
Wrought iron	0–6.1	.252	3.23
Experiment 7.			
Radium covered with 14 mm. lead.			
Bismuth	0–3.25	.533	5.44
Silver	0–3.2	.421	4.00
Copper	0–3.33	.333	3.74
Wrought iron	0–6.1	.285	3.65
Experiment 8.			
Radium covered with 10 mm. iron.			
Lead	0–3.36	.759	6.67

TABLE I. Concluded.

Substance.	Thickness, Mm.	λ	λ/d × 100
Radium covered with 20.1 mm. iron.			
Lead	0–3.36	.801	7.02
Radium covered with 30 mm. lead.			
Lead	0–3.36	.805	7.06
Bismuth	0–3.25	.656	6.69
Silver	0–3.2	.471	4.48
Copper	0–3.33	.350	3.93
Cast iron	0–10.0	.273	3.86
Experiment 9.			
Radium covered with 6.7 mm. copper.			
Lead	0–3.36	.751	6.59
Radium covered with 13.3 mm. copper.			
Lead	0–3.36	.781	6.89
Bismuth	0–3.25	.679	6.93
Cast iron	0–10.1	.261	3.69
Experiment 10.			
Radium covered with 12.2 mm. mercury.			
Lead	0–3.36	.580	5.09
Silver	0–3.2	.386	3.68
Cast iron	0–10.1	.235	3.32
Experiment 11.			
Lead	0–3.36	.554	4.86
Bismuth	0–3.25	.496	5.06
Mercury	0–2.04	.621	4.58
"	2.04–4.08	.609	4.48
"	4.08–6.12	.574	4.22
"	6.12–12.24	.554	4.07
Silver	0–3.2	.421	4.00
Copper	0–3.33	.333	3.74
"	3.33–6.69	.334	3.75
"	6.69–10.1	.329	3.69
"	10.1–13.4	.331	3.72
Wrought iron	0–6.1	.278	3.58
Cast iron	0–10.1	.263	3.72
Sulphur	0–6.9	.079	3.93
Experiment 12.			
Radium covered with 14 mm. lead.			
Lead	0–3.36	.537	4.71
Radium covered with 28 mm. lead.			
Lead	0–3.36	.484	4.24
Radium covered with 49 mm. lead.			
Lead	0–3.36	.398	3.49
Copper	0–3.36	.175	1.97
Experiment 13.			
Radium covered with 30 mm. iron.			
Lead	0–3.36	.562	4.93
Radium covered with 60 mm. iron.			
Lead	0–3.36	.539	4.72

mm. of lead, and the absorbing layer placed directly over this. The base of the electroscope was covered with only a thin sheet of cardboard. Consequently any secondary radiation which was produced from the sides of the brass box and from the surface of the absorbing material could enter the electroscope in considerable amount. The total ionization in the electroscope is thus due to several components and there is difficulty in deciding just what proportion is due to the γ rays.

A number of substances were chosen running in atomic weight from aluminum to platinum and bismuth and differing widely in density. A study of these results shows that for the high atomic weight elements the values of λ decrease with increase of thickness of absorbing layer, while for the low atomic weights the exact opposite is true. In the case of iron for example the value of λ increases from .206 to .305, while the thickness changes from 3 mm. to 30 mm. In the case of lead and mercury the value of λ seems to be approaching a more nearly constant value. Bismuth is an interesting case, since we have here an element of compara-

tively low density but high atomic weight. The very large value of λ/d here recorded is undoubtedly due to the atomic weight. It is therefore certain from these results that the stopping power is some function of the atomic weight, but does not show much change until an atomic weight of about 100 is reached. The increase in value of λ, shown for iron and copper, and which would apparently seem at first sight to indicate a softening of the rays, must be explained in some other way. It is to be noted that the absorbing layer in this experiment from which λ is calculated moves nearer and nearer to the electroscope, as the thickness is increased, thereby changing its position relative to the radium and the electroscope. This factor may have a large influence on the value of λ on account of the change in the secondary rays.

Experiment 2.—Here the radium is covered with 24.5 mm. of lead, so that the rays measured must be very hard. The values of λ for iron are now almost constant with increasing thickness. λ/d is also practically a constant for the low atomic weights, but still very large for high atomic weights.

Experiment 3.—The radium is first covered with 7 mm. of lead, and then by 15 mm. of iron. The values of λ obtained are now very large. For example copper has a value of .448, whereas in experiment 1 it only had a value of about .300.

The rays which were hardened by the lead are apparently softened by the iron.

Experiments 4 and 5.—A preliminary experiment showed that if the absorbing layer of constant thickness was placed in various positions between the radium and electroscope the values of λ did not change very much after the absorbing layer had reached a distance of about 4 cm. away from the radium. Therefore in these experiments a thin aluminum platform (*cd* in Fig. 1) was placed at a distance of 4.7 cm. from the radium. The absorbing material was placed on this platform, and any substance for hardening the rays was placed underneath.

Thus the thickness of the absorbing layer, and its relative position with regard to the radium and the electroscope, could be kept constant while the nature of the rays was changed by placing various materials underneath.

The results obtained show that for the high atomic weight elements the value of λ steadily decreases with the thickness and does not seem to be approaching a constant value. The low atomic weight atoms do not show now an increase of λ as in experiment 1, but only a slight decrease.

Experiments 6, 7, 8, 9, and 10.—These show the effect of placing various

thicknesses of different substances between the radium and the absorbing layer. It is seen that all the substances having an atomic weight over 100 harden the rays, which is shown by a decreased value of λ for all substances. Substances like iron, and copper, apparently under these experimental conditions, cause an increase in λ.

Experiment 11.—In this the base of the electroscope was covered by a plate of lead 6.6 mm. in thickness. Under this condition no secondary rays of a corpuscular nature could enter the electroscope.

The results obtained show a much more uniform character than in any of the preceding. Below an atomic weight of 100 there is not any great variation in λ/d. Above 100 the effect of the atomic weight can still be seen, though not to so great an extent as before. Below an atomic weight of 100 the variation of λ with thickness is very small, but above 100 it is still large.

Experiments 12 and 13.—These show the great hardening effect of lead and the very small hardening effect of iron.

TABLE II.

Substance.	Thickness, Mm.	Intensity, Per Cent.	λ	$\lambda/d \times 100$
Bismuth	3.00	80	.744	7.59
"	5.40	70	.660	6.73
"	8.80	60	.579	5.91
Lead	2.62	80	.851	7.53
"	4.85	70	.735	6.50
"	7.70	60	.663	5.87
Tin	6.50	80	.343	4.70
"	10.90	70	.327	4.48
"	16.60	60	.308	4.22
Cadmium	5.50	80.8	.388	4.56
Copper	6.00	80	.372	4.18
"	9.90	70	.360	4.04
"	14.5	60	.352	3.95
Iron (wrought)	8.20	80	.272	3.49
"	13.20	70	.270	3.46
"	18.90	60	.270	3.46
Aluminum	8.90	91.8	.0975	3.61
Carbon (coke)	36.0	80	.0620	4.13
"	60.0	70	.0594	3.96

We thus see from a detailed study of the foregoing results, that the value of λ can vary within wide limits depending on the experimental conditions, and the nature of the substance. It can readily be seen why it was that former experimenters gave such different results.

In Table II. are shown a number of results for different metals taken

under the following conditions. The bottom of the electroscope was covered by a plate of iron 26 mm. thick, and the radium was uncovered except for the glass tube.

The thicknesses of absorbing material are much greater than in any of the other experiments. The experimental values obtained were plotted in curves and the thickness of each substance necessary to cause absorptions of 20 per cent., 30 per cent., and 40 per cent., was taken from these curves. The curves for lead and iron are shown plotted in Fig. 2. In each case there are drawn two curves; the upper one was taken with the thin-walled brass electroscope, and the lower one with the electroscope completely surrounded by a wall of lead.

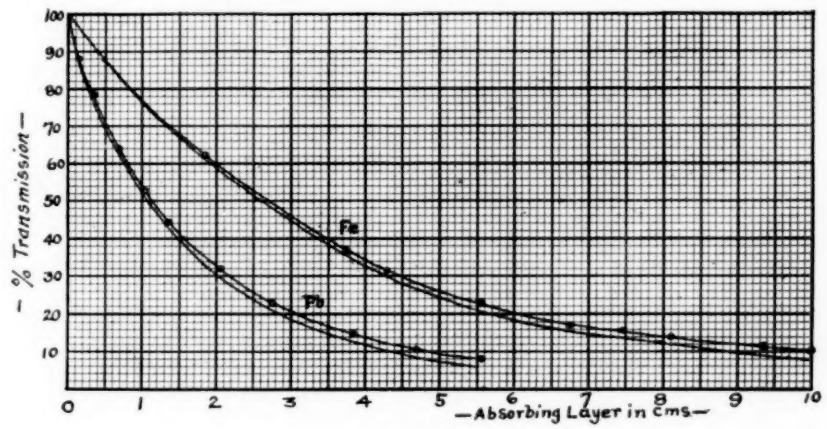


Fig. 2.

It is therefore apparent that the thickness of the electroscope has some effect on the value of the absorption coefficient; which agrees with the results of Soddy and Russell.[1] Further experiments showed that variation soon became constant and after a thickness of a half a centimeter had been reached there was no further change. This effect is ascribed by Soddy and Russell to secondary rays which are produced when the γ rays strike up surrounding objects.

An examination of Table II. illustrates very clearly the dependence of λ/d upon the atomic weight. The substances there are arranged in descending order of atomic weights and it is seen that λ/d decreases in the same order. After an atomic weight of about 100 is reached the values of λ/d do not change much.

In Fig. 3 an attempt has been made to plot the variation of λ/d with the atomic weight, taking equal absorptions of 10, 20, and 30 per cent.

[1] Loc. cit.

in each case. The curve is fairly smooth with the exception of wrought iron.

A comparison of the results in Table II. with those in Table I., when the electroscope base was covered with a thick slab of lead, shows that the values for the high atomic weight elements are very different while for atomic weights of less than 100 they are about the same. For example, the mean value of $\lambda/d \times 100$ for a lead base is 3.79, while for an iron base it is 3.83.

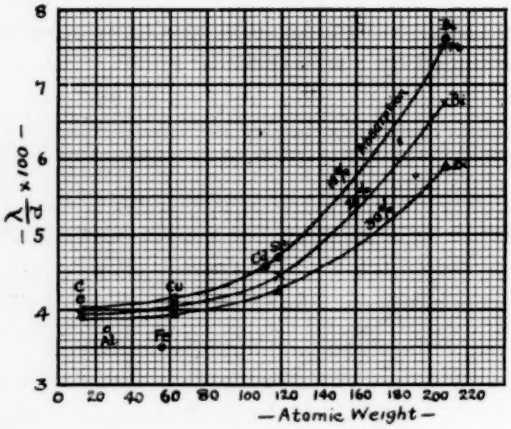


Fig. 3.

Soddy and Russell give for a number of substances below an atomic weight of 100 a value of $\lambda/d \times 100 = 3.83$, when the base of the electroscope was covered with a lead plate of 6.4 mm.

It therefore seems that the thickness or nature of electroscope base has very little effect upon the coefficient of absorption of elements of an atomic weight below 100, but above 100 the effect may be large.

Hardening Effect of High Atomic Weights.

A series of experiments were made to test the hardening effect of various substances upon the primary γ rays. In all cases the absorbing layer was 3.54 mm. of lead, placed in a fixed position relative to the radium and the base of the electroscope. The hardening material was placed between the radium and the absorbing layer.

The results are tabulated in Table III. There are two series of results shown here; one when the electroscope base was covered with the plate of lead, and the other when it was closed by the iron plate.

The first column gives the hardening substance, the second the relative amount of rays which get through the hardener, the third the proportion

TABLE III.

Hardening Substance.	Transmission Through Hardener in Per Cent.		Transmission Through Absorber in Per Cent.		λ	
	Lead Base.	Iron Base.	Lead Base.	Iron Base.	Lead Base.	Iron Base.
Nothing	100	100	81	75.4	.596	.800
Lead	65.5	63.5	82.6	80.9	.542	.607
"	45.9	43.3	83.6	82.2	.509	.553
"	25.2	30.6	84.8	83.8	.469	.500
"	—	16		86.7		.404
Bismuth	71.8	79.6	81.6	78.5	.576	.686
"	49.0	52.7	84.3	82.5	.483	.556
"	34.3	26.0	86.3	85.7	.418	.438
Mercury	68.3	65.0	82.2	80.0	.553	.633
"	34.9	44.5	84.0	81.8	.491	.571
"	19.5	25.0	85.1	85.3	.455	.452
Platinum	77.2	84.5	82.2	78.4	.554	.690
"		75.0		79.3		.659
"		66.6		80.8		.605
Tin		64.5		78.2		.698
"		52.5		79.0		.667
Antimony		73.7		78.5		.687
Cadmium	75.5	72.0	81.5	77.5	.579	.726
"	67.1	54.4	82.4	79.2	.548	.656
"	48.1	43.8	83.0	80.2	.525	.628
Copper	40		81.0		.596	
Iron	40		81.4		.582	
Nickel		70		75.5		.794

of these which get through the lead absorber, and the final values of λ calculated from column three.

Three general groups are shown here; those having a high atomic weight, such as lead, bismuth, platinum and mercury; those having a medium atomic weight, such as tin, antimony, and cadmium; and those having a low atomic weight, such as copper, iron, and nickel.

It can readily be seen that the hardening effect is a function of the atomic weight, and is quite large for high atomic weights. Cadmium shows quite an appreciable effect, but very little effect can be detected

with coppper or iron. That this hardening effect is not due to the absorption is easily seen, for copper after absorbing 60 per cent. of the rays, shows very little hardening, while a very small absorption by lead is sufficient to harden the rays.

For any one substance the hardening effect increases with the absorption.

That the rays have been actually changed in penetrating power after they have passed through a hardening substance is shown by the fact that the coefficient of absorption of a substance like iron, or copper, is changed when the rays have previously passed through lead, although iron and copper themselves show very little hardening effect. For example λ for copper is .343 when no hardener is present, but .302 when the rays have passed through a thickness of 3 cm. of lead.

Soddy and Russell[1] give some results on this hardening effect which are about the same as in this paper. In Table IV. are given some results taken from their paper.

TABLE IV.

Thickness of lead in cm.	0	.124	.249	.373	.64	1.24	2.50	3.75
λ for zinc.	.325	.322	.309	.300	.291	.272	.265	.258

THE ABSORPTION BY LIQUIDS.

A number of experiments were made on the absorption of the rays by liquids. The liquids were placed in a cylindrical glass vessel, and the rays passed through the glass bottom before entering the liquid.

In most of the cases the liquids were placed in a large glass vessel upon the aluminum platform, but for a few of the liquids the quantity available was not large enough to fill the vessel to a sufficient height to cause appreciable absorption. In these cases a narrower vessel was used, placed beneath the platform, and by using water under both conditions the values for any liquid could be corrected to what they would have been in the large vessel.

In Table V. the results are shown for a number of pure liquids and for three characteristic aqueous solutions. These were taken without any covering of the base of the electroscope.

The results show about the same characteristics as for the solid substances. The value of λ/d is practically a constant for all substances which do not contain any high atomic weight elements. The effect of high weight atoms is clearly shown by the lead nitrate solution in its large value of λ/d. The value of λ/d for water seems larger than one would expect, also for the alcohols.

[1] Loc. cit.

TABLE V.

Pure Liquids.

Substance.	Thickness, M m.	λ	$\lambda/d \times 100$
Water	0–20.4	.0462	4.62
Ethyl alcohol	0–20	.0352	4.40
Ether	0–20	.0326	4.46
Ester	0–20	.0417	4.50
Amyl bromide	0–20	.0544	4.50
Carbon tetra bromide	0–14.8	.1144	3.73
Benzene "	0–20	.0607	4.04
Ethelene "	0–18	.0870	3.95
Propyl "	0–9	.0617	4.54
Iso amyl "	0–19	.0685	4.10
Meta brom anilene	0–20	.0731	4.56
Ethyl iodide	0–12	.0860	4.52
Methyl "	0–8	.0907	4.12
Iod benzene	0–20	.0783	4.28
Aqueous Solutions.			
Lead nitrate	0–20	.0754	5.57
Potassium iodide	0–20	.0760	4.81
Copper chloride	0–20	.0594	4.12

TABLE VI.

Pure Liquids.

Substance.	Thickness, Mm.	λ	$\lambda/d \times 100$
Water	0–18.4	.0465	4.65
"	0–32.7	.0472	4.72
Amyl alcohol	0–18.3	.0355	4.37
Amyl bromide	0–18.3	.0497	4.11
Iod benzene	0– 6.5	.0740	4.05
Chloroform	0–18.4	.0603	4.06
Sulphuric acid	0–18.4	.0757	4.11
Aqueous Solutions.			
Lead nitrate	0–18.4	.0652	4.82
" "	0–32.7	.0636	4.68
Potassium iodide	0–18.4	.0658	4.17
" "	0–26.5	.0653	4.13
Copper chloride	0–18.4	.0609	4.22
" "	0–32.7	.0612	4.25
Cadmium iodide	0–18.4	.0549	4.28
Nitric acid	0–18.3	.0497	4.19
Bismuth trichloride	0–18.4	.0465	4.33

Any particular molecular grouping has no effect upon the absorption, which seems to obey the additive law in the same manner as β rays.

The stopping power of a liquid per unit of mass depends only on the kind, and relative number, of the constituent atoms.

In Table VI. are shown a similar set of results when the base of the electroscope was covered by the lead plate.

The constancy of λ/d for low atomic weights is here even better than in Table V. The mean value of a number having atomic weights below 100 is .0412, which is a little higher than the same for the solids.

In Table VII. are shown results on two solutions at different concentrations.

TABLE VII.

Solution.	Concentration by Mass.	Density.	λ	$\lambda/d \times 100$
Water	0	1.00	.0459	4.59
Lead nitrate	.10	1.082	.0534	4.93
" "	.20	1.162	.0592	5.09
" "	.30	1.241	.0640	5.16
" "	.40	1.319	.0670	5.08
" "	.50	1.393	.0739	5.30
Potassium iodide	.20	1.133	.0510	4.50
" "	.40	1.257	.0606	4.82
" "	.60	1.364	.0640	4.70
" "	.80	1.461	.0681	4.66
" "	1.00	1.551	.0698	4.50
" "	1.20	1.621	.0718	4.43
" "	1.40	1.691	.0756	4.47

SUMMARY AND DISCUSSION OF RESULTS.

1. It has been shown that the coefficient of absorption of the γ rays, as determined in the ordinary way by the ionization method, depends greatly upon the experimental disposition of the various parts of the apparatus. The chief factors are: the position of absorbing layer relative to the radium and the electroscope; the thickness and nature of the electroscope base; and the previous screening of the rays. This to a great extent explains the diversity of results furnished by past experimenters on this subject. The results of the author in this respect are in general agreement with those of Soddy and Russell.

2. When the best arrangement suitable for the given case has been chosen, it can be shown that the " stopping power per unit mass " (λ/d) is a function of the atomic weight of the absorbing material, increasing very slowly with the atomic weight until an atomic weight of about 100 is reached, and then more rapidly.

3. When the rays first pass through any substance before reaching the absorbing layer they are hardened. This hardening is a function of the

atomic weight and is large for high atomic weight atoms. It is not proportional to the amount of absorption.

4. It has been shown that the absorption of the γ rays by liquids is a function of the atomic weights of the constituent atoms. In other words the absorption of the γ rays is an additive property of the atoms, and is independent of molecular grouping.

5. The values of λ/d obtained in this work are in general agreement with those of Soddy and Russell, wherever the experimental conditions are the same.

Excluding the initial soft rays, the value of λ/d for all substances below an atomic weight of 100 may be taken for approximate calculations as .0410. If there has been previous hardening, or if the absorbing material has an atomic weight above 100, the value of λ/d will depend on the conditions of the case.

At the present time there are two theories advanced to account for the phenomena which occur when γ or X rays penetrate an absorbing layer. One of these considers the rays to consist of a succession of ether pulses while the other considers them to be negative corpuscles which in some unknown manner have had their charge neutralized by equal quantities of positive electricity. These two theories differ materially in their consideration of the so-called secondary radiations. On the ether pulse theory the secondary rays derive their energy from the atoms from which they come; but on the corpuscular theory they have the same energy that they had as constituent parts of the primary beam.

Barkla[1] divides the secondary X radiation into two kinds: one which is merely the primary beam deflected and has the same properties; the other which is characteristic of the atom bombarded and is called the homogeneous fluorescent radiation. This latter kind increases in penetrating power with the atomic weight and can only be excited by a primary beam of greater penetrating power. Besides these two radiations there are at all times a large number of electrons liberated.

The ether pulse theory was devised to account for X ray phenomena originally, whereas the corpuscular theory was based on experiments dealing chiefly with γ rays.

At the present time most physicists are convinced of the similar nature of X and γ rays, so that any theory which explains one must of necessity include the other. Yet there is a possibility that they are not quite the same. Until one has examined X and γ rays under exactly the same conditions as to hardness one cannot finally decide the question. X rays have a number of peculiarities which γ rays have not exhibited as yet.

[1] Barkla, Phil. Mag., 1911.

For example in the case of X rays, some substances, notably silver, show selective absorption varying with the hardness of the ray. A similar phenomenon is not present in the case of γ rays. Unfortunately the hardest X rays yet examined are still a long way from the γ rays.

The author is at present engaged in trying to bridge over this gap, and the results will be incorporated in a paper to be published soon.

The hardening of the γ rays in the case of radium by successive layers of any material has been explained in the past by the theory that the rays are at the start heterogeneous in penetrating power and that the atoms of high atomic weight offer more rapid absorption to the soft rays than do the lighter atoms. The soft components are thus cut down very rapidly leaving the residue more and more penetrating.

Soddy is of the opinion that the γ rays of radium are homogeneous, but in his results gives several cases of hardening. If the rays are homogeneous it is difficult to see how they can be hardened, according to the corpuscular theory, where each is a separate entity unchangeable in nature.

My thanks are due to Mr. Edward Lorentz, of the Department of Physics, for his assistance during the course of this work.

UNIVERSITY OF CINCINNATI,
January, 1912.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

VARIATION OF ELECTRICAL RESISTANCE WITH TEMPERATURE.[1] IV. OXIDES.

BY A. A. SOMERVILLE.

SINCE the effect of temperature upon the electrical resistance of many of the so-called conductors found in a laboratory has been studied, the writer has found an interesting field for work in the investigation of the behavior of the non-conductors at ordinary conditions, if those materials are subjected to higher temperatures.

Fig. 1.

Also as most metals and alloys oxidize when heated in the air, it seems just as natural to investigate the products of oxidation as to try to prevent oxidation by excluding oxygen.

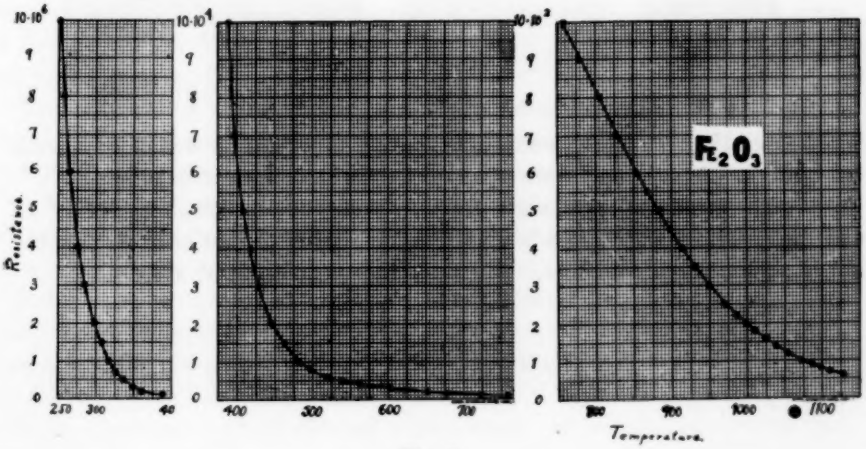


Fig. 2.

For these reasons the work has been begun of measuring resistance as a function of temperature of the oxides.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

As a rule, oxides at room temperature are non-conductors. As the temperature is increased the resistance of an oxide decreases until at 1000° C. it may be classed as a conductor.

The oxide in the form of a powder is placed in a porcelain or quartz tube (Fig. 1) 1 cm. in diameter and 10 cm. in length. Nickel rods are machined to fit into the ends of the tube against the ends of the column of oxide which is about 5 cm. long. Two to four pounds pressure is put on the ends of the nickel rods in order to effect constant contact with the oxide. The oxide in its containing tube is heated in a tubular resistance furnace. Temperature is measured with a platinum resistance thermometer. Electrical contact is made with the nickel rods for measuring the resistance of the oxide.

A column 1 cm. in diameter and 5 cm. in length, of iron oxide, Fe_2O_3, has a resistance of 10,000,000 ohms at a temperature of 250°. As the temperature

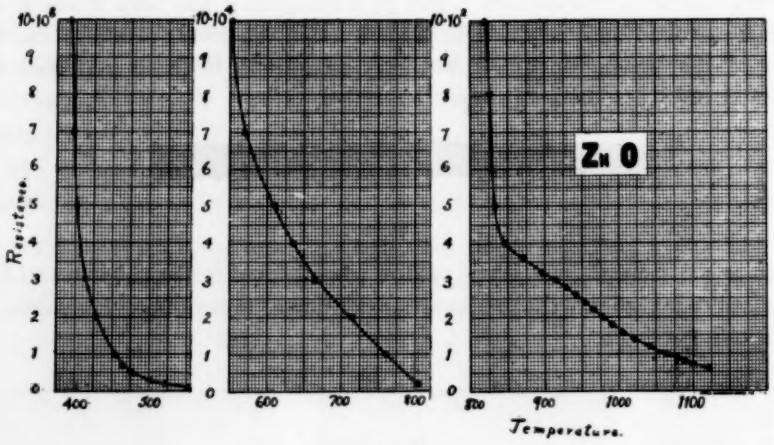


Fig. 3.

is increased the resistance decreases according to a relation similar to a parabolic curve. At a temperature of 1125° C. the resistance is 60 ohms.

A similar column of zinc oxide, ZnO, measures 10,000,000 ohms at 385° C. and the resistance varies in a body somewhat similar to that of iron oxide but not in such a regular manner. The resistance is 30 ohms at 1125° C.

CORNELL UNIVERSITY,
ITHACA, N. Y.

International Candle-Power Measurements.[1]

By Clayton H. Sharp.

In 1903 the writer took a set of seasoned incandescent lamps to the National Physical Laboratory in London, to the Laboratoire Central d'Électricité in Paris and to the Physikalisch-Technische Reichsanstalt in Charlottenburg, in each of which laboratories the candle-power of these lamps was measured. The results were studied with a view of determining the status as to the standards of luminous intensity used in the countries in question. In this country the Bureau of Standards had adopted a ratio of 1 Hefner = 0.88 candle-power which value of the standard corresponded to that which had been in use in the electrical industry for a good many years previous and which had been maintained by the use of standardized incandescent lamps. It was found by the measurements referred to that the standard used in this country was in reasonably good conformity with the German standard, using this ratio. It appeared further that the value of the bougie décimale as used in France was very nearly equal to our candle, while in England the standard was not sufficiently well defined, although it apparently did not differ very much in value from the standard in this country.

Since 1903 numerous international comparisons using incandescent lamps have been very carefully carried out by the Bureau of Standards, which organization has also been able to bring order out of confusion through the adoption of the so-called "International Candle." The matter of standards therefore being presumably put on a satisfactory basis, it was deemed interesting to repeat the operation of carrying a set of seasoned lamps once more to these various laboratories and to extend the comparison somewhat in view of more recent developments in lamp manufacture. This the writer did during the past summer.

Four lamps were taken, of the drawn wire tungsten type. At each point where a spire of the filament passed a support, it was welded fast to the support, so that all loose or variable contacts in the lamp were avoided. These lamps, after having been seasoned, were standardized at two different voltages, (1) the voltage at which the color of the light was the same as the color of the carbon lamp as ordinarily operated, and (2) the voltage at which the color of the light was the same as that of the tungsten filament lamp as ordinarily operated. The second voltages were substantially 50 per cent. higher than the first. The reason for taking two voltages was that it was known that the color difference introduced in comparing a carbon with a tungsten lamp caused an error in photometry which differed with different individuals and which was by no means negligible. On account of this the Electrical Testing Laboratories had already adopted a series of tungsten lamp standards for regular work and it was desired to see how closely the value of these standards would be represented by the measurements obtained in foreign laboratories.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

The results of the measurements are given in the following table. The values given by the Reichsanstalt are reduced to candle-power by multiplying them by the present nominal ratio of the Hefner to the candle, namely, 0.9.

	Lamp No. 3,892 at 69.2 Volts.		Lamp No. 3,893 at 69.8 Volts.		Lamp No. 3,894 at 69.5 Volts.		Lamp No. 3,895 at 69.15 Volts.		Averages.	
	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.
E.T.L.	6.0	−0.50	7.0	−0.28	8.5	−0.47	8.5	+0.47	7.5	−0.18
P.T.R.	6.075	+0.75	7.10	+1.14	8.65	+1.17	8.57	+1.18	7.60	+1.13
L.C.E.	6.0	−0.50	6.95	−1.00	8.5	−0.47	8.35	−1.30	7.45	−0.85
N.P.L.	6.06	+0.50	7.05	+0.43	8.565	+0.29	8.50	+0.47	7.54	+0.35
B.S.	6.0	−0.33	7.0	−0.28	8.5	−0.47	8.4	−0.71	7.48	−0.45
Mean	6.03		7.02		8.54		8.46		7.51	

	Lamp No. 3,892 at 106.8 Volts.		Lamp No. 3,893 at 100.4 Volts.		Lamp No. 3,894 at 105.75 Volts.		Lamp No. 3,895 at 105.2 Volts.		Averages.		Ratio.
	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	H.V.C.P. / L.V.C.P.
E.T.L.	31.0	0	27.97	+0.25	42.0	+0.48	42.0	+0.48	35.74	+0.42	4.76
P.T.R.	31.6	+1.93	28.4	+1.79	42.9	+2.56	42.95	+2.75	36.45	+2.42	4.80
L.C.E.	30.5	−1.61	27.5	−1.43	40.7	−2.63	40.6	−2.86	34.82	−2.16	4.68
N.P.L.	30.9	−0.32	27.7	−0.72	41.75	−0.12	41.65	−0.36	35.50	−0.25	4.71
B.S.	30.8	−0.64	27.8	−0.36	41.6	−0.48	41.6	−0.48	35.45	−0.39	4.74
Mean	31.0		27.9		41.8		41.8		35.59		4.74

It will be seen that the various laboratories are in close agreement in their evaluation of these lamps as far as their determination at the ordinary color of carbon lamps is concerned. The greatest deviation from the mean is shown by the Reichsanstalt, namely, 1.1 per cent., which is not a surprising deviation in view of the fact that the Reichsanstalt measurements are all carried back to the Hefner lamp itself, whereas in the case of the other laboratories, measurements are carried back to a series of standardized incandescent lamps. Therefore the 1.1 per cent. may represent a variation of the Hefner lamp from its mean value at the particular time it was used in connection with the measurements in question. With respect to the other deviations it may be said that they are so small that they illustrate the extreme closeness with which the standards are kept and the great care with which the photometric measurements were carried out. Such precision in international candle-power determinations was a thing not to be thought of only a few years ago.

With respect to the measurements of the lamps at the whiter color, the situation is not so good. Here considerable deviations come in which are evidently due to different individual evaluations of lights of different color. The variations run from + 2.4 per cent. to − 2.2 per cent. However, it is to be noted that in only three laboratories, namely, the National Physical Labo-

ratory, the Bureau of Standards and the Electrical Testing Laboratories, had special attention been given to the establishment of a well defined line of white light standards and that among these laboratories the variations are less than the ordinary error of precision photometric work.

The above contribution therefore goes to show that international precision as well as precision in a single laboratory has been established in photometric measurements, and to emphasize the fact that a series of standards of only one color will not do. Different series of standards are required in precision work in order that the color of the standard may not differ too much from the color of the source of light which is under measurement.

ELECTRICAL TESTING LABORATORIES,
NEW YORK.

MAGNETIC REACTIONS PRODUCED BY A COPPER DISC ROTATING BETWEEN THE POLES OF A MAGNET.[1]

BY F. G. BENEDICT AND W. G. CADY.

THESE observations were made on the revolving disc of a "bicycle ergometer" belonging to the Nutrition Laboratory of the Carnegie Institution of Washington, but the results are generally applicable to all conductors moving in the neighborhood of magnet poles. The machine experimented upon is a stationary bicycle having a disc of copper about 40 cm. in diameter and 6 mm. thick mounted in place of the rear wheel. A calorimetric method is used to determine the amount of energy expended (chiefly in the form of eddy currents in the disc) when the latter is rotated between the poles of a fixed electro-magnet. These calibrations have shown that the rate of production of heat does not increase with increasing angular velocity as rapidly as would be expected from elementary considerations. When, however, the self-inductance of the current paths in the disc is taken into account, the peculiar form of the calibration curves is fully explained.

Starting with the assumptions that the angle of lag of the currents is directly proportional to the angular velocity, and that the consequent demagnetizing effect of the currents is proportional to this angle and to the intensity of the currents, the equation

$$\frac{\sigma}{k} = \omega^2 \frac{\varphi}{\varphi'}$$

is deduced, in which σ is the specific resistance of the disc, ω the angular velocity, φ the resultant magnetic flux, and φ' the counter-flux due to the eddy currents; k is a constant of proportionality.

Some preliminary observations were made with a ballistic galvanometer, in order to learn the extent to which the total flux was reduced when the disc was set in rotation. A magnetic survey of the whole region between and near

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

the magnet poles was then made, using a bismuth spiral to measure the magnetic induction at different points. The results when exhibited in the form or curves show that the magnetic distortion and the weakening of the magnetic field increase very greatly with increasing angular velocity, and that the equation above is approximately satisfied for all speeds up to 364 revolutions per minute of the disc.

A simple computation shows that the total intensity of the currents in the disc is of the order of 2,000 amperes.

The Electron Theory of Phosphorescence.[1]

By Chester A. Butman.

A THEORY of phosphorescence has been more completely developed, which explains in a consistent manner the usual phenomena observed. The principal points in the theory are as follows:

1. There is a direct relation between the photoelectric effect and phosphorescence.

2. The component which gives the photoelectric effect is *not* the one which emits the light.

3. The spectra of the phosphorescent material are due to a *selective emission* of light.

4. If the selective emission is caused by the incident light alone, fluorescence occurs. However, if the fluorescence is maintained by *collisions* of the particles which have received electrical charges due to the electrons ejected by the photoelectric material, the phenomenon of phosphorescence is observed.

5. In the case of either fluorescence or phosphorescence the electronic systems of the atom emitting the light must be brought to their *critical values* of oscillating energy before an emission of light will occur.

6. It is necessary that phosphorescent substances be good insulators so that the charges produced by the photoelectric component will not disappear too quickly.

7. The effect of red and infra-red light is to dampen the oscillations of the electronic systems emitting the light, causing them to fall below the critical value, thus extinguishing the phosphorescent light.

8. The different phosphorescent bands can be generated separately, because the electronic systems in the atom are most easily brought to their critical value by particular wave-lengths.

According to the theory (CaO + BiO + S + Flux) is phosphorescent because BiO is capable of giving a selective emission of light in the visible spectrum. The BiO is diluted with CaO, which is an insulator and is transparent for the visible spectrum. The sulphur is photoelectric with visible light and furnishes the supply of electrons. The oscillating electronic systems in the Bi,

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

which cause the selective emission bands, having been brought to their critical value by the incident light, are maintained in that state by the collision of the electrified particles with the Bi. Inasmuch as the material has high insulating qualities the charges (ejected photoelectrically from the sulphur) remain for some time, hence the luminosity also remains. However, the duration of the luminosity is also dependent on the ease with which the oscillating systems are maintained at their critical value.

Feb. 21, 1912.

On the Expansion of Alternating Current E. M. F.'s in a Fourier's Series.[1]

By Geo. R. Olshausen.

In many text-books and treatises on alternating currents the statement is made that practically all alternating current machines generate E.M.F.'s which, when plotted in rectangular coördinates as a function of the time, present a certain symmetry with respect to the axis of abscissæ when it is taken as the time axis. The further statement is then made that, owing to this symmetry, an expansion of the E.M.F. function in a Fourier's series contains only the odd harmonics. As I do not know of any general proof of the last statement, the following one may be of interest.

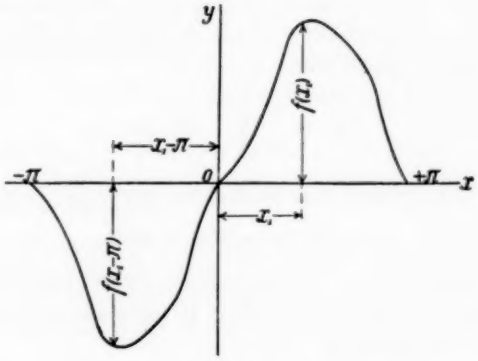


Fig. 1.

The symmetry referred to is of the type shown in Fig. 1, in which the E.M.F.'s are the ordinates and the products $2\pi ft$ the abscissæ, t being the time and f the frequency. We have therefore

$$y = f(x) = \text{E.M.F.},$$
$$x = 2\pi ft.$$

If we develop

$$y = f(x)$$

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

in a Fourier's series, we have in general

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + \cdots + a_n \cos nx + \cdots + b_1 \sin x + b_2 \sin 2x + \cdots + b_n \sin nx + \cdots,$$

where the coefficients, as is well known, have the following values, viz.:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} f(x)dx,$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos nx\,dx,$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin nx\,dx.$$

We shall now show that a_0 and all coefficients for which n is even vanish on account of the assumed symmetry.

If we let x_1 be a particular value of x, such that

$$0 < x_1 < \pi,$$

we have, as seen from the figure,

$$f(x_1) = -f(x_1 - \pi).$$

It is easily seen that for each positive value x_1 of x there exists a corresponding negative value $x_1 - \pi$ for which $f(x)$ has the same absolute value.

Consider now the constant term

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} f(x)dx.$$

The elements of this integral can be divided into pairs, such that

$$f(x_1)dx = -f(x_1 - \pi)dx,$$

dx being always positive since $f(x)$ is a single-valued function. The elements of this integral therefore annul each other in pairs and a_0 is consequently zero.

Next, consider the elements

$$f(x_1)\cos nx_1 dx \quad \text{and} \quad f(x_1 - \pi)\cos(x_1 - \pi)dx$$

of the integral in

$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos nx\,dx.$$

$f(x_1)$ and $f(x_1 - \pi)$ have the same absolute value but have opposite signs. $\cos nx_1$ and $\cos n(x_1 - \pi)$ have the same value when n is even, but are of equal magnitude and of opposite sign when n is odd. The elements under consideration have therefore the same absolute value for all values of n, but are of opposite sign when n is even. In the latter case the sum of each pair of such elements is therefore zero, and consequently a_n vanishes.

In precisely the same manner it can be shown that the coefficients b_n are zero only when n is even.

For curves of the assumed symmetry the development of $f(x)$ into a Fourier's series therefore contains only the sine and cosine terms of the odd multiples of x, so that we have

$$f(x) = a_1 \cos x + a_3 \cos 3x + a_5 \cos 5x + \cdots + b_1 \sin x + b_3 \sin 3x + b_5 \sin 5x + \cdots.$$

The form of the expansion of $f(x)$ for other forms of symmetry may be determined in a similar manner. For example, in Fig. 2 we have a curve which

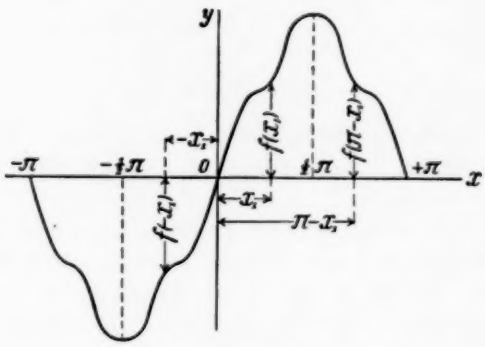


Fig. 2.

is symmetrical with respect to the origin and at the same time the two halves of the curve are also symmetrical to ordinates passing through the points

$$x = -\frac{\pi}{2} \quad y = 0 \quad \text{and} \quad x = \frac{\pi}{2}, \quad y = 0$$

respectively. We have therefore

$$f(x_1) = -f(-x_1) = f(\pi - x_1).$$

It can easily be shown that on account of these two conditions the expansion of $f(x)$ will only contain the sines of the odd multiples of x, so that

$$f(x) = b_1 \sin x + b_3 \sin 3x + b_5 \sin 5x + \cdots.$$

If $f(x)$ is symmetrical with respect to the y axis and each half of the curve is also symmetrical as above, so that

$$f(x_1) = f(-x_1) = f(\pi - x_1),$$

the expansion will in general contain a constant term and the cosines of the even multiples of x. We have then

$$f(x) = a_0 + a_2 \cos 2x + a_4 \cos 4x + \cdots.$$

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